MECHANICAL

Problems,

ADAPTED TO THE COURSE OF READING PURSUED

IN THE

UNIVERSITY OF CAMBRIDGE:

COLLECTED AND ARRANGED

FOR THE USE OF STUDENTS.

LONDON:

PRINTED BY R. GILBERT, ST. JOHN'S SQUARE:

AND SOLD BY G. B. WHITTAKER,

AVE MARIA LANE;

AND J. AND J. J. DEIGHTON, T. STEVENSON, AND NEWBY, CAMBRIDGE.

1828.

TO THE

REV. WILLIAM WHEWELL, M.A. F.R.S.

FELLOW OF TRINITY COLLEGE,

THE FOLLOWING PAGES,

DESIGNED TO PROMOTE THE STUDY

OF

THAT BRANCH OF PHILOSOPHY

WHICH HAS BEEN ILLUSTRATED AND IMPROVED

BY

HIS "ELEMENTARY TREATISE,"

Are Inscribed;

IN ADMIRATION OF HIS TALENTS,

AND

RESPECT FOR HIS INDUSTRY.

PREFACE.

It has been confessed by most of those who have been engaged in the laborious task of tuition, that the principles of any branch of mathematical or philosophical science are more readily comprehended, and more easily imprinted upon the student's memory, when their application to the solution of problems has been pointed out and explained. With the intention of contributing in some measure to this desirable end, the following collection of Problems, which were originally intended to appear in a very different form, are now printed; and it is hoped that, even in this state, they may be found of essential service to such of the younger members of our University, as have not the benefit of a private tutor's superintendence, and access to those sources of assistance which such superintendence usually supplies.

In the following pages, a considerable number of the more easy forms have been placed at

the beginning of each section, or subdivision: and the examples have been so disposed, as gradually to lead the student to the solution of such as are more complicated in their form, or involve greater difficulties in the investigation. With the exception of a few, it will be found that no principles are involved, which are not contained in Mr. Whewell's "Elementary Treatise of Mechanics," whose arrangement however has not been entirely adopted.

Many of the Problems which are here offered have been proposed at public and private examinations in the University, and several may be found scattered elsewhere: but the solutions which have accompanied the latter, have been in many instances tedious and perplexed: they are therefore now proposed to exercise the ingenuity of the Cambridge students; who in general are so trained as to combine quickness of invention with accuracy of investigation, extensiveness of reading with solidity of judgment.

LONDON,
January 30, 1828.

ERRATA.

Page 6, line 14, *for* 1 =, *read* = 1.

19, — 23, *for* $4P$, *read* $4P^2$.

58, — 19, *for* end, *read* ends.

In the Press, and speedily will be published,

A

COLLECTION OF PROBLEMS

IN THE

DIFFERENT BRANCHES OF PHILOSOPHY,

ADAPTED TO THE COURSE OF READING PURSUED
IN THE UNIVERSITY OF CAMBRIDGE.

MECHANICAL PROBLEMS.

SECTION I.

1. If two equal forces, inclined at any angle, act upon a body: prove that the resultant or compound force bisects that angle.

2. At what angle must two equal forces act, that the resultant may be equal in magnitude to either of them separately. Determine the angle also, when the resultant is n^{th} part of either.

3. There are two forces represented by 3 and 4. At what angle must they act so that the resultant may be represented by 5 ?

4. Two equal forces are inclined at an angle of 30°; find the magnitude of the resultant. Determine the magnitude also of the resultant, when they act at angles of 45°, 60°, and 120°: and in each case also when the forces are in the proportion of 2 : 1.

5. Two forces act upon a plane, and produce the same effect; one is applied at an angle of 30°, and the other perpendicularly. Determine

the ratio of the forces. Determine the ratio also, when one is applied at an angle of 45°.

6. Compare the effects of two equal forces acting upon a plane, one perpendicularly, and the other at an angle of 60°.

7. Two equal forces act upon a plane, one perpendicularly, and the other obliquely, and their effects are as 2 : 1. Determine the angle at which the latter acts.

8. If the angle at which two given forces act be diminished, the resultant is increased. Determine the angle, when the resultant is a mean proportional between its greatest and least values.

9. Resolve a given force into two others, 1. which shall always be equal; 2. whose sum or difference shall always equal a given quantity; 3. which shall contain a given angle; 4. which shall act at a given angle, and whose sum shall be given; 5. which shall act at a given angle, and whose difference shall be given.

10. Given the difference between the resultant and either of two equal forces acting at a right angle; determine the forces by a geometrical construction.

11. If a stream runs at the rate of two miles an hour; determine at what angle the course of a boat must be inclined to it, so that it may pass directly across it, supposing the boat to be rowed at the rate of four miles an hour.

12. In a gale of wind of eight knots an hour, a ship also runs eight knots an hour, and *appa-*

rently to those on board within four points (45°) of the wind. In what direction, with regard to the wind, is the ship really going?

13. A person travelling on a road that tended directly west, at the rate of 6 miles an hour, observed that the wind appeared to strike him from the N. W.; but having occasion to stop, he then found that it actually came from a point 10° more to the north. Required the velocity of the wind.

14. If two ports bear N.E. and S.W. of each other, distant 50 leagues; and a S.E. current runs between them at the rate of three knots an hour: how must a vessel from the northern port shape her course, so that running at the uniform rate of five knots an hour, she may reach the other port in the least time?

15. Two chords, AB, AC, of a circle, represent two forces; one of them, AB, being given, find the position of the other, when the resultant is a maximum.

16. If SA, the least distance in an ellipse, be $\frac{1}{3}$ SM (the greatest), and SA be taken to represent one force; then SB, the mean distance, will compound with SA, the least possible force: and this resultant will be a mean proportional between SM and SA.

17. Three equal forces act upon a plane, and their effects are as 5, 4, 3. The first acts perpendicularly; required the inclinations of the others to the plane.

18. Three equal forces act upon a body, each

in a direction perpendicular to the plane of the other two. Find the resultant, and compare it with either.

19. If three forces, A, B, C, act upon a given point, and keep it at rest; given the magnitude and direction of A, the magnitude of C, and the direction of B: to determine the magnitude of B, and the direction of C.

20. If three forces act upon a body, shew that it cannot remain at rest unless the forces act in the same plane, and meet in a point, and unless the direction of each passes through the angle formed by the directions of the other two.

21. If three forces be represented by the three adjacent sides of a rectangular parallelopiped: prove that the sum of the squares of the sines of the angles, which each force makes with the resultant, is a constant quality.

22. If α, β, γ, be the angles which the resultant of three forces, acting at right angles to one another, makes with each of them respectively, then will $\cos. 2\alpha + \cos. 2\beta + \cos. 2\gamma = -1$.

23. AB, AC, AD, are three lines making angles of 120^0 with each other; the point A is acted upon by pulling forces in AB and AC, which are as 3 and 4, and by a pushing force in DA, which is as 5. Find the force which will keep it at rest.

24. A point in the vertex of a right angled triangle is solicited by a number of forces, represented in magnitude and direction by lines drawn to equidistant points in the base. Required the magnitude and direction of the resultant.

25. ABC is a right angled Isosceles triangle, and three *equal* forces act *in* the lines AB, BC, CA. Determine at what point of the plane ABC, produced if necessary, a force must be applied to keep it at rest, and what must be its magnitude and direction.

26. If forces proportional to the sides of a quadrilateral figure be applied perpendicularly at their middle points, they will keep one another in equilibrio.

27. If two known equal and parallel forces act in the same direction at the opposite angles A and C of a given parallelogram $ABCD$; determine the two parallel forces, which, acting at the angles B and D, will keep the parallelogram at rest.

28. If two parallel forces act in the same direction on the opposite angles A and C of the parallelogram $ABCD$, and a third force act on the point B in the direction of the diagonal BD; find the magnitude and point of application of a fourth force, which will keep the parallelogram at rest.

29. Given the directions of four forces in different planes, acting upon a given point, and keeping it at rest. Determine their magnitudes.

30. If the sides $a, b, c, \ldots . n$, of a polygon, represent the magnitudes and directions of (n) forces which keep a point at rest, prove that $n^2 = a^2 + b^2 + c^2 + \ldots . . - 2\,(ab. \cos. a, b + ac. \cos. a, c + bc. \cos. b, c + \&c.)$.

31. If any number of forces, p, q, r, &c. in

different planes, acting on a point, make the angles α, β, γ, &c. with the resultant, then will $p.\cos.\alpha + q.\cos.\beta + r.\cos.\gamma +$ &c. be a maximum.

32. If a body be kept at rest by three parallel forces, shew that they are in the same plane, and any two of them are to each other inversely as their distances from the third.

33. A given force being applied at a point P within a tetrahedron, it is required to resolve it into four others applied at the angular points A, B, C, D; and to shew that if the lines drawn from these points through P, meet the opposite faces in a, b, c, d, respectively, $\frac{Pa}{Aa}+\frac{Pb}{Bb}+\frac{Pc}{Cc}+\frac{Pd}{Dd}1=$.

SECTION II.

1. ONE pound is weighed at the ends of a false balance, and the sum of the apparent weights is $2\frac{1}{2}$ lbs. Determine the ratio of the arms.

2. The same weight is weighed at the two ends of a false balance, and it is observed that the whole gain is $\frac{1}{n}^{\text{th}}$ part of the true weight. Determine the distance of the fulcrum from the middle point of the balance.

3. If one of the arms of a false balance be longer than the other by $\frac{1}{n}^{\text{th}}$ part of the shorter, and when it is used the weight is put into one scale as often as into the other. What will be the gain or loss per cent?

4. If the difference of the lengths of the arms of a lever be (a) inches, and the same weight weighs (w) pounds at one end, and (w) ounces at the other; determine their lengths.

5. If in a false balance a body weighs (w) at one end, and (w') at the other; find the centre of suspension, and its distance from the centre of the beam.

6. Suppose an uniform straight lever to have some weight, and at one end a weight suspended equal to that of the lever. Where must the fulcrum be placed, that there may be an equilibrium? Find the place of the fulcrum when the suspended weight is (n) times that of the lever.

7. A lever, 3 feet in length, weighs 3 lbs; what weight on the shorter arm will balance 12 lbs on the longer, the fulcrum being one foot from the end?

8. A weight of 20 lbs. placed 3 feet from the fulcrum of a lever, whose weight is 30 lbs, balances the other arm. Required the length of the lever.

9. Two given weights, A and B, are suspended at the extremities of an uniform straight lever of given weight (w), and given length (l). Determine the distance of the fulcrum from either end, when there is an equilibrium.

10. A cylindrical piece of oak timber, of given length, being suspended at a given distance from one of its ends; required a weight (w) to be attached to the other end, to keep it parallel to the horizon. What is the diameter of its end, its density being (d)?

11. A beam (m) feet long, balances itself upon a point at $\frac{1}{n}^{\text{th}}$ of its length from the thicker end; but when a weight (w) is suspended at the other end, the prop must be moved (a) feet towards it, to maintain the equilibrium. What is the weight of the beam?

12. On a lever of uniform density, every inch of which weighs (w) ounces, a weight of (W) pounds is suspended at a given distance from the fulcrum placed at one extremity. Determine the length of the lever, so that the whole may be supported by the least possible power acting in an opposite direction at the other extremity.

13. If a piece of timber, 17 feet long, be rested on a prop placed at the distance of 4 feet from one end; it is found that one cwt. at that end would be balanced by 12 pounds at the other; and that if the places of the weights are exchanged, the prop must be 8 feet from the other end. Determine the weight of the timber, and the place of the prop on which the timber would balance without the weights.

14. A weight (w) is placed at the middle point of a straight lever, two weights (P) and (W) at its extremities. When there is an equilibrium, the fulcrum is at a distance from (P) equal to $\frac{1}{n}^{\text{th}}$ of the length of the lever; but if the places of (w) and (W) be changed, it is at a distance of $\frac{1}{m}^{\text{th}}$ of the length. Determine (P) and (W).

15. At the extremity of the shorter arm of a lever of uniform density, there is placed a weight, which is to the weight of the lever, as the difference between the lengths of the arms is to their sum. Determine the ratio of the arms.

16. If the arms of a lever (without weight) be equal, and inclined to each other at an angle of 120°; determine the ratio of two weights (P) and (W), which, suspended from their extremities, are in equilibrio. Determine the ratio also when the inclination is 150°: one of the arms in each case being horizontal.

17. The weights of the arms of a bent lever are (m) and (n). At what angle must they be placed, that the lever may be in equilibrio when the shorter arm is parallel to the horizon?

18. The arms of a lever are perpendicular to each other and equal: the weights (w) and (w') are suspended at each end. What is the position of the lever when they sustain each other? Find it also in the case where $w : w' :: 1 : \sqrt{3}$.

19. Two weights (w) and (w') are suspended from the extremities of the arms of a straight lever without weight, which are as 3 : 5. Determine the ratio of $w : w'$, when the former acts at an angle of 60°, and the latter at an angle of 45°.

20. Two weights (w) and (w') are applied at two given points of a given lever, at angles (θ) and (θ'). Determine the place of the fulcrum, that there may be an equilibrium, and find the pressure upon it.

21. Two scales are suspended from the ends of a straight lever whose arms are as 3 and 4; and an iron bar of 20 lbs. weight is laid upon the scales, and will just reach from one to the other. What weight is required to preserve the equilibrium?

22. A cylindrical rod whose length is 7 feet, and weight 14 lbs, is placed on a fulcrum at the distance of 3 feet from one of its extremities. How must another cylinder of equal weight, whose length is 4 feet, be suspended by two equal and parallel strings fixed to its extremities, from the cylindrical rod, so that the whole may be in equilibrio?

23. Two cylindrical iron rods whose diameters are 4 and 3, and length of each 4 inches, are joined together so as to form one straight lever.

Where must the fulcrum be placed that the lever may be balanced, when a weight equal to twice the weight of the smaller cylinder is hung at the extremity of the thicker rod?

24. A lever whose arms are (a) and (b) inches respectively, is kept at rest by a given weight (w) acting at the extremity of the shorter arm, and two other weights (P) and (Q), of which (P) acts at the extremity of the longer arm, and (Q) at (c) inches from it. Determine (P) so that (Q) may be an harmonic mean between the weights at the extremities.

25. There are (n) weights a, b, c, &c. in geometric progression, and (a) placed at A one extremity of a lever balances (b) placed at B the other extremity. Prove that a weight equal to the ($n-1$) first weights if placed at A will balance a weight equal to the ($n-1$) last weights at B.

26. The weights of the scales (A) and (B) of a balance being known and unequal, and the beam considered as without weight; it is required to determine the position of the fulcrum, so that a given weight (P) may balance another given weight (W) when placed in the scale (A), and also a given weight (W') in the scale (B).

27. To determine the length of a lever (supposed without weight) which may be divided into two arms A and B, such that $A^m - B^m$ may be equal to $A^n \times B^n$; and two given weights (W) and (W') when placed at their extremities may keep each other in equilibrio.

28. In a steelyard, if the weight increase in

arithmetic progression, the divisions of the scale will be at equal intervals; and if each of these intervals be equal to the shorter arm, the moveable weight will be equal to the difference of the arithmetic progressionals.

29. If a lever be graduated in harmonic progression, the weights of bodies may be ascertained by means of a moveable fulcrum and a known weight; the graduation beginning at that extremity of the lever at which the body to be weighed is placed.

30. In the Swedish steelyard, the body to be weighed and the constant weight are fixed at its extremities, and the fulcrum is moveable. If then (n) bodies in arithmetic progression are weighed in succession, and the two first are (w) and (w'), determine the distance of the fulcrum from either end when the last body is suspended.

31. If a scale E be suspended from A the extremity of a straight lever ACB; prove by the resolution of forces, that the effect of the scale when pushed out of its vertical position by the action upwards of a rod EA applied at A, is the same as if the scale were suspended from a point P, where the perpendicular from E meets the arm CA produced.

32. If the scales of a balance be suspended from the extremities of its corresponding arms, in a perpendicular direction, by means of an inflexible bar, it is found that a person weighed in such a balance may protrude the arm nearest him in

any part of it without destroying the equilibrium. Required a theoretical demonstration.

33. Suppose a person suspended in a balance to act upwards by means of a rod against a point in the arm of the lever opposite to that in which he is suspended: will his weight be increased or diminished by this action, and in what ratio?

---

34. A lever, at whose extremities two weights (*P*) and (*W*) hung by threads, balance each other, is made to revolve about its fulcrum; shew that if the threads be equal, (*P*) and (*W*) describe concentric circles; if unequal, similar segments of circles.

35. At what height must the end of a given rope be fixed from the base of a pillar, so that a given power acting at the other end may be most effective to overturn it?

36. Given the weight of a wheel whose radius is 4 feet, to determine what power acting in an horizontal direction will draw it over a square stone 2 feet in height.

37. If the plane on which a carriage moves, and the line of draught be both horizontal, shew that the power requisite to overcome a *small* given obstacle varies inversely as the square root of the radius of the wheel.

38. The force requisite to draw a wheel over a small obstacle varies nearly as the square root of the height of the obstacle. Required proof.

39. If (W) be the weight sustained by the wheels of a carriage; what is the force necessary to keep it at rest upon a road inclined at a given angle to the horizon, the line of draught being nearly parallel to the road?

40. On a horizontal plane is an immovable obstacle (a) inches high, over which a heavy laden carriage is to be drawn by the least possible force. Determine the diameter of the wheels; the height of the horse's breast to which the traces are applied being (b) feet, and the distance thence to the axle (c) feet.

41. If the arms of a wheelbarrow be elevated above their usual position, it will require a less force to push it horizontally. Determine the elevation of the arms when the barrow moves horizontally with the greatest ease, by the force of its own weight.

42. Explain the construction of Lord Stanhope's lever; and find the ratio of $P : W$.

43. The beam of a false balance being of uniform density and thickness; it is required to shew that the lengths of the arms are respectively proportional to the differences between the true and apparent weights.

44. AP, BW, are the directions of two parallel forces (P) and (W) which sustain each other on the equal arms of the bent lever ACB Draw CD perpendicular to AB, and CE parallel to AP or BW; then

$$P+W : P-W :: \text{Tang. } \tfrac{1}{2} ACB : \text{Tang. } DCE.$$

45. If two forces (P) and (W) sustain each

other on the arms of a bent lever PCW, and act in directions PA, WA, which form the sides of an Isosceles triangle PAW. Shew that if AC be joined, and produced to meet PW in E,

$$P : W :: WE : EP.$$

46. If AP, BQ represent two forces acting, at given angles with the horizon, on the equal arms AC, CB of a straight lever ACB whose fulcrum is C; and Pp, Qq be drawn perpendicular to ACB; shew that there will be an equilibrium when $Ap + Bq$ is a maximum.

47. The weight (W) is connected by the string AW to a vertical pole AB, and is forced out of its vertical position by a force exerted horizontally in the direction PW. To what point of the *pole* must an equal and opposite force be applied, that the whole effect of this force to break the pole may be counteracted?

48. A body is suspended from a given point in the horizontal plane by a string of known length which is thrust out of its vertical position by a rod (supposed without weight) acting from a given point in the plane, against the body. Shew that the tension of the string varies inversely as the tangent of the inclination of the rod to the horizon.

49. The density of a lever of given length varies as the $(n)^{\text{th}}$ power of the distance from one extremity by which it is suspended. A given weight P attached to the other end, and acting perpendicularly, keeps the lever horizontal. The lever when P is removed would vibrate (m) times

in (t) seconds. Determine the weight of the lever and the index (n).

50. (a) and (b) are the arms of a straight lever which turns on an axis, the radius of which is (r). A weight (P) would maintain the equilibrium acting perpendicularly at the distance (a), if there were no friction; but a weight (p) must be added to it in order to overcome the friction. Determine the proportion of the friction to the pressure.

SECTION III.

1. In a combination of wheels and axles, each of the radii of the wheels is to each of the radii of the axles :: 5 : 1. If there be 4 wheels and axles, determine what power will balance a weight of 1875 pounds.

2. In a combination of wheels and axles, in which the circumference of each axle is applied to the circumference of the next wheel, and in which the ratios of the radii of the wheels and axles are 2 : 1, 4 : 1, 8 : 1, &c. there is an equilibrium when the power is to the weight as $1 : p$. Determine the number of wheels.

3. In what position will the weight which a given power supports by means of a wheel and square axle, be a mean proportional between the greatest and least weights supported on the same machine?

4. Two weights (P) and (W) are supported on a wheel and axle, (P) by a string passing round the wheel (W) by a moveable pulley whose strings are parallel; one of which, as (P) descends, winds *on* the wheel, and the other *off* the axle. Determine the ratio of $P : W$ in case of equilibrium; and point out the advantage of this system.

5. In a system of wheels moveable by teeth and pinions; having given the ratios of the num-

ber of teeth in each wheel and pinion; determine the number of times the $(n)^{th}$ wheel turns round its axis whilst the first performs (m) revolutions.

6. Suppose a wheel A turns another wheel B, on whose axis is a wheel C that turns a wheel D. Determine the least number of teeth in each of these four wheels, so that A and D may make equal revolutions, when D revolves round C in 365 turns of A.

7. Two given weights being attached to given points in the circumference of a wheel; find the position in which the greatest weight will be supported on the axle.

8. When wheels act by teeth working in one another, the force of one upon the other will remain constant, if the line which is drawn perpendicular to the surfaces of both the teeth at the point of contact, pass continually through the same point of the line which joins the centres of the wheels.

---

9. What power will sustain 40 pounds over four moveable pullies?

10. In a system of pullies, where each pulley has a separate string passing over it, and fastened to the weight; having given P and W, determine the number of moveable pullies.—Suppose P and W to be as $1:63$, what is the number?

11. In a system of pullies, where each pulley hangs by a separate string; given (P) and (W)

and the number of pullies; determine the weight of each pulley.—Also in the system where the strings are attached to the weight.

12. In a system of pullies, where each pulley hangs by a separate string, the weight is suspended from the centre of the lowest pulley, and the power applied by means of a string passing over a pulley A. Supposing the number of moveable pullies to be 4, and the angles formed by the directions of the strings at each of the pullies B and C to be 120°, and at each of the pullies D and E to be 60°; determine the ratio of $P : W$ when there is an equilibrium.

13. In the single moveable pulley, if (W) be supported by two powers (P) and (P') whose directions touch the pulley in A and B; $W : P$ or $P' :: \sin. \theta : \sin. \frac{1}{2}\theta$, where θ is the angle contained by PA and $P'B$.

14. If a body (W) resting upon a string between two fixed pullies be balanced by two equal bodies (P) and (P) hanging on opposite sides; the distance of (W) from the horizontal line joining the pullies is equal to $\frac{W \times D}{\sqrt{4P - W^2}}$, where D is $= \frac{1}{2}$ the distance of the pullies from each other.

15. In the single moveable pulley, when (P) descends, find the curve which is the locus of (W).

16. In a system of (n) moveable pullies, where each pulley hangs by a separate string, and the strings are parallel, if the weights of the pullies, reckoning from the one nearest to (W), increase in a geometric progression whose common ratio

is 2; find the equation between (P) and (W) when there is an equilibrium, (w) being the weight of the lowest pulley.

17. In a system of pullies in which the strings are parallel, and each string is attached to the weight, will the weight of the pullies increase or diminish the mechanical advantage? And if in this system there are (n) moveable pullies, the weights of which, reckoning from the one nearest to (P), increase in a geometric progression whose common ratio is 2, what will be the equation between (P) and (W) when there is an equilibrium, (w) being the weight of the first pulley?

18. A power (P) is applied at the extremity of a string which passes over a fixed pulley A, under a moveable pulley D, over a fixed pulley B, under a moveable pulley E, and is fixed at C. To the centre of D is attached a string which passes under a moveable pulley F, and is fixed to the centre of E. At the centre of F the weight is applied. Find the ratio of $P : W$, and the pressures at A, B, C, when there is an equilibrium.

19. (P) and (W) are two weights connected by a string passing over a fixed pulley; (P) hangs freely and (W) slides along an oblique line. To determine the place where these two weights are in equilibrio.

20. If with a horizontal lever which keeps two weights in equilibrio, there be combined a single moveable pulley, the mechanical advantage will be exactly doubled. Required proof.

21. A parabola has its axis horizontal, and a body (P) on the curve is connected by a string passing over a fixed pulley at the focus of the parabola, to a weight (W) suspended freely. Shew that in order that (P) may rest in all positions, (W) must be reciprocally proportional to the corresponding ordinate.

---

22. A power of 1 lb acting parallel to a plane supports a weight of 2 lbs. Determine the angle of inclination of the plane.

23. A weight of 40 pounds acting parallel to the length sustains another of 56 pounds on an inclined plane, whose base is 340 feet. Determine the height and length of the plane.

24. A given weight (P) draws another (W) up an inclined plane by means of a thread running parallel to the plane, and the force stretching the string is $(\frac{1}{n})^{\text{th}}$ part of the descending weight. Determine the plane's inclination to the horizon.

25. Compare (P) and (W) when they are in equilibrio on a plane, whose inclination is 30°; the power being supposed to act parallel to the base of the plane.

26. Determine the ratio of $P : W$, when a body is supported on an inclined plane of 45°, and the direction of inclination of (P) to the plane is 45°. —Also when the power acts parallel to the plane, and the angle of inclination is 60°.

27. A weight (P) hanging freely raises ano-

ther weight (W) up an inclined plane by means of a string not parallel to the plane: determine the tension of the string.

28. A weight (P) hanging freely supports an equal weight (W) on an inclined plane by means of a string passing over a pulley below the plane: determine the position of equilibrium.

29. If the height of a plane be equal to half the length of the base; compare the weight sustained by a power acting parallel to the length with that sustained by it when acting parallel to the base.

30. If a body be supported on an inclined plane, first by a power parallel to its length, and then by a power parallel to its base; compare the pressures on the plane.

31. If a weight (P) descending vertically draw another weight (W) up a given inclined plane AB by means of a string passing over a pulley fixed at a given altitude above it; having given the ratio of $P : W$, determine the point where (W) must rest upon the plane, so as to be in equilibrio with (P).

32. If in the case of the last problem, AB be supposed to be a quadrantal arc; determine the position of (W) in the case of equilibrium; (P) and (W) being known.

33. Two planes of equal altitudes are inclined at angles of 60°, and 45°; determine the ratio of $P : W$ when they are supported upon them, by means of a string passing over the common vertex and parallel to the planes.

34. If the planes are of unequal altitudes and inclinations, and the weights are supported on them, by means of a connecting string passing over a pulley placed in a vertical line passing through the upper extremities of the planes: determine the ratio of $P : W$.

35. If PO be the direction of a force which sustains a body P on an inclined plane AC, and PD be drawn at right angles to PO meeting the base AB in D. Shew that AD, PD, and PA, will represent the quantities of the weight, power, and pressure.

36. ACD is a bent lever, whose arms AC, CD, are as $1 : 2$, and inclined to each other at an angle of 120°. Also when CD is parallel to the horizon, a weight (P) hanging freely from A counterbalances another weight (W) suspended from D, and partly supported on an inclined plane whose elevation is 30°. Determine the ratio of $P : W$.

37. What number of pullies hanging by separate parallel strings must be applied to a weight of 96 pounds, so that a power of one pound may sustain it on an inclined plane whose height is one-third of its length, supposing the power to act parallel to the length?

38. P and W balance on two inclined planes which have a common altitude. If they be put in motion shew that their vertical velocities will be inversely as their quantities of matter.

39. A globe of given weight is supported between two planes inclined to the horizon at the

respective angles of 30° and 60°. Compare the weights sustained by the planes with each other and with the whole weight. What will be the ratio, if the angles be 45° and 60°?

40. A cubical solid rests in equilibrio between two inclined planes containing an acute angle. Determine its position, and compare the pressures on the supporting planes. Prove also that the ratio of these pressures is independent of the figure of the solid.

41. Two bodies (A) and (B) connected by a string passing over the common section C, support each other upon the inclined planes CD, CE. If a circle be described through the points C, A, B, and the bodies without changing their position be connected by any other string passing over a point in the circumference of the circle, prove that the equilibrium will still remain.

42. Find the equation of equilibrium on the inclined plane, when friction proportional to the pressure is taken into the account.

---

43. Compare the force on the back with the sum of the resistances on the sides of a Scalene wedge; the forces acting at any given angles.

44. In an Isosceles wedge, the resistances act an angle of 45°, and the angle at the vertex of the wedge is 60°. What is the ratio of the resistances to the force on the back?

---

45. The distance between two contiguous threads of the screw is 2 inches, and the arm to which (P) is applied is 20 inches. Determine the ratio of $P : W$ when there is an equilibrium.

46. If the thread of a screw be inclined at an angle of 30°, and the radius of the cylinder be (r), the length of the lever which turns the screw being (l). Determine what power will sustain a weight of (n) lbs by it.

47. In the endless screw, (W) is kept at rest on the axle of the wheel by a power (P) acting at the extremity of the arm BC, which is at right angles to the axis of the screw. Having given the radii of the wheel and axle, the distance between the threads and the length of the arm BC; determine the ratio of $P : W$.

SECTION IV.

1. If two bodies approach each other in a right line; what must be the ratio of their velocities, that their centre of gravity may remain at rest?

2. If three forces, represented in magnitude and direction by three lines, GA, GB, GC, keep the body G at rest; the place of the body will be the centre of gravity of the triangle, formed by joining the extremities of the lines.

3. What must be the form of the triangle, that the centre of gravity may be the centre of the circumscribing circle?

4. What must be the form of the triangle, that the centre of gravity may be the centre of the inscribed circle?

5. The three lines drawn from the three angles of a triangle, and bisecting the opposite sides, intersect each other in a point; which is the centre of gravity of the triangle?

6. If the sides of a triangle taken in order be cut proportionally, prove that the triangle formed by joining the points of section will have the same centre of gravity as the original triangle.

7. Given the distances of three bodies from their centre of gravity, and the angles at the centre. To compare the bodies.

8. If a straight line be drawn through the

centre of gravity of a triangle, to meet two sides, and the third side produced; the rectangle under the segments of this line, measured from the centre of gravity on one side of it, is equal to the sum of the rectangles under the same two segments, and the segment on the other side of the centre of gravity.

9. If the sides of the triangle *ABC* be bisected in the points *D, E, F;* then the centre of the circle inscribed in the triangle *DEF* is the centre of gravity of the perimeter of the triangle *ABC.*

10. From *P* to any number of fixed points, *A, B, C,* &c. draw *PA, PB, PC,* &c., so that $PA^2+PB^2+PC^2+$ &c. may be equal to a constant quantity. Shew that *P* will always lie in a spherical surface, whose centre is the centre of gravity of the points *A, B, C,* &c.

11. If a circle or a sphere be described about the centre of gravity of any number of bodies, and any point be taken in the periphery of this circle or surface, shew that the sum of each body into the square of its distance from this point, is a constant quantity.

12. The sum of the squares of the distances of the centre of gravity of any number of equal bodies, from the centre of gravity of each, is equal to the sum of the squares of the distances of the centres of gravity of these bodies, taken two and two, divided by the number of bodies.

---

13. Find the centre of gravity of three equal bodies, placed at the three angles of an Isosceles right angled triangle.

14. If triangles be inscribed in a given circle; what is the locus of their centres of gravity, when their base is a given chord.

15. What will be the locus of the centres of gravity of all triangles of a given area, having their vertices in a given point, and their vertical angles the same?

16. To find the centre of gravity of a quadrilateral figure, which has two adjacent sides equal, and its other two adjacent sides also equal.

17. Find the centre of gravity of a quadrilateral figure, two sides of which are parallel.

18. If (a) and (b) are the two sides of a quadrilateral figure that are parallel to each other: shew that the centre of gravity will divide a perpendicular to those sides into two parts that are to each other as $2a+b : 2b+a$.

19. Find the centre of gravity of the three squares described on the three sides of a right angled triangle.

20. If two equal Isosceles triangles, one of whose sides is to the base :: 5 : 8, be placed so that perpendiculars from their vertices to their bases form a right angle. Required the distance of their common centre of gravity from the point where they touch.

21. If two Isosceles triangles, whose altitudes are (h) and (h'), stand upon the same base, and on the same side of it; determine the distance of

the centre of gravity of the area included between *the sides* of the two triangles, from the vertex of the greater.

22. The distance of the centre of gravity of a regular polygon, inscribed in a sector of a circle, from its centre, is equal to two-thirds of the distance of the centre of gravity of the arc.

23. Shew that in any polygon, the sum of the squares of the distances of the centre of gravity from the angular points is the least possible.

24. Find the centre of gravity of a parabola, cut off by an ordinate to *any* diameter.

25. Find the position of the centre of gravity of the quadrant of a circular area.

26. Find the position of the centre of gravity of the area of a semiparabola.

27. Find the centre of gravity of a straight line, in which the weight of each particle varies as the distance from one end.

28. Find the centre of gravity of a triangle, in which the weight of each particle is the same in the same line drawn parallel to the base; but in different parallels varies directly as the distance from a line passing through its vertex parallel to the base.

29. Find the centre of gravity of a semicircle, in which the weight of each particle varies as its distance from the centre.

30. The distance of the centre of gravity of a cycloid from the vertex is equal to $\frac{7}{12}$ of the axis. Compare from this, the contents of the solids generated by its revolution round the base, and a tangent at the vertex.

31. Given the length of a curve; to determine its nature, when its centre of gravity is most remote from the axis.

32. Find the centre of gravity of a portion of a given paraboloid cut off by any plane.

33. If R and r be the radii of the greater and lesser end of the frustum of a paraboloid, whose height is h; determine an expression for the position of the centre of gravity.

34. Find the centre of gravity of the segment of a sphere:—and of a spheroid.

35. Find the centre of gravity of a triangular pyramid; and shew in what ratio it divides the line joining any of the angular points, and the centre of gravity of the opposite side.

36. The centre of gravity of a triangular pyramid is the same with that of four equal bodies placed at its corners.

37. If the middle points of any two edges of a triangular pyramid, which do not meet, be joined: shew that the middle point of the connecting line is the centre of gravity of the pyramid.

38. If from the four angles of a pyramid there be drawn lines to its centre of gravity; shew that a point placed there shall be kept at rest by forces represented by the four lines.

39. If the pyramid formed by drawing lines from the angles of the base of a triangular pyramid, to a point within the pyramid, be taken away: find the centre of gravity of the remaining figure.

40. If two cones, whose altitudes are (h) and

(h'), be described on the same base, but on opposite sides: find the centre of gravity of the solid.

41. If from a cone whose altitude is (h), another having the same base, but an altitude (h') be taken away; find the centre of gravity of the remaining solid.

42. If from a sphere whose radius is (r), another whose radius is (r'), touching the other internally, be taken away: find the centre of gravity of the remaining figure.

43. Find the centre of gravity of a hemisphere, in which the weight of each particle varies as its distance from the centre.

44. If from a given paraboloid the greatest cylinder possible be taken away. Determine the centre of gravity of the remaining solid.

45. Find the centre of gravity of the solid generated by a quadrant of a circle through one fourth of a revolution about the radius.

46. Find the centre of gravity of the surface generated by an arc of a circle during $(\frac{1}{n})^{\text{th}}$ of a revolution round a radius passing through one of its extremities.

47. Find the centre of gravity of the solid generated by a circular area during $(\frac{1}{n})^{\text{th}}$ of a revolution round a radius, which is one of its boundaries.

48. Find the centre of gravity of a wedge, of which the sides are cut into the form of a parabola, the flat surfaces being exactly similar and equal.

49. Find at what distance from the vertex of

the catenary, its centre of gravity is; without the aid of logarithms.

50. Given $ax^n = y^2$, the equation to a curve: determine the exponent (n) and the length of the curve, when the distance of the centre of gravity of a solid formed thereby, from the vertex, is equal to $\frac{25}{34}x$.

51. Determine the distances of the centres of gravity of a right cone and of a spherical surface, from the vertex of each; and hence deduce the distance of the centre of gravity of a spherical sector from the centre of the sphere.

52. Given that the distance of the centre of gravity of a cone from the vertex is $\frac{3}{4}$ of the axis: find the centre of gravity of a conic frustrum, the radii of whose bases are (R) and (r), and altitude (h).

53. Given that the distance of the centre of gravity of an area from the vertex is $(\frac{1}{n})^{th}$ part of the abscissa; determine the distance of the centre of gravity of the solid generated by the same area revolving round its axis.

54. If the distance of the centre of gravity of the surface of a solid from the vertex is equal to half the abscissa: determine the nature of the curve by which the solid was generated.

---

55. How high can a wall 5 feet thick, and inclined at an angle of 60°, be built without falling?

56. The slant height of a wall is equal to twice

its horizontal thickness: determine its inclination when it will be just supported.

57. An oblique cylinder of given substance is placed with one of its ends on a horizontal plane, the inclining side making an angle of 60° with the horizon, its height and the diameter of its base are (a) and (b) feet respectively. Determine the diameter of the greatest sphere of the same substance, which the cylinder will sustain without falling; supposing it to hang freely from the edge of the top.

58. If a cone, whose altitude and diameter of its base are as 2 : 3, be placed on a horizontal plane, and a ball, whose diameter is (d), being suspended from its vertex by means of a string, be just sufficient to overturn it: determine the dimensions of the cone; the distance of the centre of the ball from the vertex being (a) feet, and the specific gravity of the cone and ball as $1 : s$.

59. An equilateral cone, whose diameter is (d), stands upon a horizontal plane. Required the solidity of a part to be cut off, so that the remaining part may just support itself from falling.

60. $ABCD$ is a quadrilateral figure, of which the two shorter sides, AB, BC, are equal, as also the two longer, AD, DC, and the angle ABC is a right angle. What is the greatest length of the side AD, that the figure may stand on the base AB, on a horizontal plane, without oversetting?

---

61. A cube is placed on an inclined plane, whose angle of elevation is 50°. Will it roll or slide?

62. An equi-angular prism is placed upon an inclined plane with its axis parallel to the horizon, and is just supported. Determine the plane's inclination.

63. Determine the solidity of the greatest cone of given diameter, which can be supported on a plane whose inclination is 30°; and find the highest point in the slant side, where a weight may be placed without overturning it.

64. Find the inclination of a plane on which a regular polygon of (n) sides will just be supported.

65. Find the greatest inclination of a plane upon which a given elliptic cylinder, whose axis is horizontal, can be supported.

66. What is the least slope down which a regular hexagonal prism could roll?

---

67. Among all the axes passing through the centre of gravity of a triangle in its own plane; determine that for which the momentum of inertia is a maximum or a minimum.

---

68. Three equal bodies move in the same time through the three sides of a triangle. Determine the motion of their centre of gravity.

69. Two bodies descend down two sides of a triangle, and they are inversely as the sides of the triangle: in what proportion does the path of the centre of gravity divide the angle at the vertex?

70. Let two equal bodies, from the vertex of an Isosceles right angled triangle, move uniformly along the sides. Compare the space described by the centre of gravity with the space described by either body.—Compare also the spaces when the vertical angle is 120°.

71. Two bodies move uniformly and in the same time from the right angle of a triangle to the different extremities of the hypothenuse, the acute angles being 60° and 30°. Determine the relative magnitudes, when the path of the centre of gravity bisects the right angle:—and also when it is perpendicular to the hypothenuse.

72. Three bodies, *A*, *B*, *C*, which are as the numbers 1, 2, 3, move separately in the same time round the three sides of an equilateral triangle. Determine the space passed over by their common centre of gravity.

73. Upon a given base, *AB*, and in a given plane, two equilateral triangles, *ABC*, *ABD*, are described. If the triangle *ABC*, be made to revolve in the same plane, round the angular point *A*, what path will have been described by the common centre of gravity when the sides *AC*, *AD*, coincide?

74. Four bodies, *A*, *B*, *C*, *D*, represented by the numbers 1, 2, 3, 4, are placed at the angles of a given square, and move separately over the four

sides in the same time. Determine the length of the path described by the centre of gravity.

75. Four bodies set out at the same time from the angular points of a trapezium, and move in the direction of the sides taken in order. Now the bodies, as also their velocities, which are uniform, are in the sub-duplicate ratio of the sides which they are describing. How will their centre of gravity move?

76. If one body rests in the centre of a circle, and another describes the circumference; determine the path of the centre of gravity.

77. If one body describes the circumference of a circle, and another be placed at a given point without it: determine the path described by the centre of gravity.

78. If two bodies set out from the extremity of the diameter of a circle, and one describes the diameter whilst the other describes a semicircle, what will be the path of the centre of gravity?

79. Two equal bodies move at the same instant from the same extremity of the diameter of a circle, with equal velocities, in opposite semicircles. Determine the path described by the centre of gravity.—Determine the path also, if the bodies are unequal. Also, if one remaining at rest, the other describes the circumference.

80. Two bodies (A) and (B) are at one extremity of the axis major of an ellipse, and $A=3B$; now (B) remaining at rest, (A) moves round the periphery. Determine the path of the centre of gravity.

81. Let two bodies setting out from the extremity of the axis major, describe two ellipses, which have the same axis, and equal areas round the centre in equal times. Determine the path described by the centre of gravity.

---

82. Two bodies begin to fall from the same extremity of the vertical diameter of a circle, one through the chord of 60°, and the other through the chord of 120°. Compare their magnitudes, when the line, which their centre of gravity describes, bisects the angle made by their directions.

83. Two bodies begin to descend from the same extremity of the vertical diameter of a circle, one down the diameter, and the other down the chord of 30°. Find the ratio of their magnitudes, when their centre of gravity moves along the chord of 120°.

84. Two equal bodies begin to descend at the same instant through the two chords of a semicircle, drawn from a point in the circumference to the extremities of its diameter, which is perpendicular to the horizon. Determine geometrically the path described by their centre of gravity; prove that its motion is uniformly accelerated, and compare the force with which it is accelerated with that of gravity.

85. Two bodies, whose magnitudes are given, begin to descend at the same instant, one down the length, the other down the height of an inclined plane. Determine the path of their common centre of gravity.

86. A body (A) is let fall from a given point at the same time that another body (B) is projected from the same point, along a horizontal plane. Determine the path of the centre of gravity.

87. Two equal bodies begin their descent at the same instant down a plane inclined at an angle of 30° to the plane of the horizon, and in directions perpendicular to the common section of the two planes. One of them, after descending half way, is reflected by a perfectly elastic plane, inclined at an angle of 45° to its course. Determine the nature and dimensions of the path traced out by their common centre of gravity.

88. A weight (P) upon an inclined plane is supported by a weight (W) hanging freely, the string being parallel to the plane. Shew that if they be moved into any other position, the centre of gravity moves in a horizontal line.—If (P) and (W) be not in equilibrio, what will be the path of the centre of gravity?

89. Two weights, (P) and (W), sustain each other on two inclined planes, which have a common altitude by means of a string parallel to the planes. Shew that if they be put in motion, their centre of gravity describes a right line parallel to the horizon.

90. Let two bodies of known magnitudes be projected upwards at the same time with the velocities (m) and (n). Determine the height to which their common centre of gravity will rise, and the time of ascent.

91. A body of (2) lbs weight is projected with

a velocity of (20), at an angle of 60°; another of (3) lbs, at the same time, is projected with a velocity of (25), at an angle of 30°. Trace the motion of the common centre of gravity; find the height to which it rises, and the distance at which it strikes the horizontal plane.

92. If any number of balls, of given weight, are projected at the same instant, in given directions, with given velocities; find the height of their common centre of gravity, after a given time; and the highest point to which it will rise.

93. Two given spheres, imperfectly elastic, are projected from the extremities of the same horizontal line with equal velocities, and at equal angles, but in opposite directions. How will the centre of gravity move before and after the collision?

94. If a given weight, placed upon an inclined plane, be connected, by means of a string passing over a fixed pulley, with another given weight hanging freely; determine the path of their centre of gravity; the relative position of the pulley and plane, and also the length of the string being given.

95. If two bodies be projected at equal distances from a plane to which they are attracted, by a force varying inversely as the cube of the distance, and with velocities which are inversely as the sines of the angles which the directions of projection make with the plane; prove that their common centre of gravity will describe a conic section.

SECTION V.

1. If two forces balance, which are inclined at angles (α) and (β) to the arms (a) and (b) of a straight lever *not* attached to its fulcrum; then $a : b :: \text{tang.}\ \beta : \text{tang.}\ \alpha$.

2. Shew that if (α) be the angle at which the arms of a common balance are inclined, and that (P) and (W) when in equilibrio sustain the lever at an angle (θ) with the horizon,

$$\text{Tang.}\ \theta = \frac{P - W}{P + W} \cdot \text{Cot.}\ \tfrac{1}{2}\alpha.$$

3. AC, CB, are the equal arms of a straight lever whose fulcrum is C; to C a heavy arm CD is fixed perpendicular to AB. Prove that when different weights are suspended from the extremity A, the tangents of the inclinations of CD to the vertical will be proportional to the weights.

4. Two weights (P) and (W) are placed at the extremities of a lever, (P) hanging by a thread. Determine where the fulcrum must be placed, so that the tension of the thread may be a maximum.

5. A cone (a) inches long is suspended by a string fastened to its middle point C: and a body hung on at the distance of (b) inches from C on the narrower side is balanced by a weight of (c) pounds suspended at the thicker end: but removing the body one inch nearer to C, the (c) pounds weight on the other side is moved within

(d) inches of C before the cone will rest in equilibrio. Determine the weight of the body.

6. A heavy cone of given dimensions stands inverted on its vertex, its axis making an angle of a^0 with the horizon. What must be the magnitude and direction of the least power, which applied at the centre of its base will just sustain the cone in that position?

7. If the cone stand on the edge of its base, the axis making an angle of a^0 with the horizon; what must be the direction and quantity of the least power applied to its vertex that will just sustain the cone in that position?

8. A given conical beam rests with its vertex against a smooth vertical wall, and the base is sustained by a known weight fastened to a string which passes over a fixed pulley. Determine the position of the beam when at rest, and the pressure against the wall.

9. A prop, (a) feet long, standing perpendicularly to a horizontal plane, sustains the greater end of a beam of given dimensions and substance in the form of a conical frustum, the other end resting on the plane. How much less does the prop bear, when the beam is in this position, than when the lesser end is elevated till the axis of the frustum is parallel to the horizontal plane?

10. Given the dimensions of a conical frustum; determine to what degree of elevation it must be raised, with the greater end downwards, so that a vertical prop of given length may support $(\frac{1}{n})^{th}$ part of the weight of the frustum.

11. A beam BC is supported in a given position by means of a prop DE of given length insisting on the horizontal beam AB. Determine the position of the prop, so that the force whereby the beam AB tends to break, may be to the whole force it can sustain at E in the least ratio possible, the thickness of the beam being every where the same.

12. AC, BD, are two beams whose weights are given, moveable in a vertical plane about the fixed points A and B; and BD rests upon AC as a prop. Determine the position of equilibrium in general;—and also when AC is equal to AB.

13. Prove that when a system is in equilibrio the centre of gravity is the highest or lowest possible. And hence deduce the position of equilibrium of the two equal beams AC, BD, which, revolving in a vertical plane round the points A and B in the horizontal line AB, support a given weight on a string joining their summits.

14. If the points C and D (in the preceding problem) be connected by a string of given length which passes over a fixed pulley; determine the position of the beams when they are in equilibrio, the pulley and beams being in the same vertical plane.

15. Determine the position in which two given beams of uniform density will rest, when they are placed on a horizontal plane with their lower extremities opposed to each other, and their other extremities supported on two parallel vertical planes.

16. If a quadrant of given weight and radius, moveable about its centre (which is fixed) in the same vertical plane, be supported with its upper edge parallel to the horizon by a string attached to the higher extremity of the arc, passing over a pulley, and bearing a weight at the other end just sufficient to maintain the equilibrium. Determine the ratio of this weight to the weight of the quadrant; the pulley being placed at a given distance from the quadrant, and at a given altitude above the level of the horizontal edge.

17. A string is fixed to the extremities of the axis of a cone of given weight and dimensions, and is suspended over a tack void of friction: determine the position in which the cone will rest, and the tension of the string.

18. If a cone of given weight and dimensions be suspended by the vertex about which it is freely moveable; and two given weights, fastened by cords to a ring in the circumference of the base, pass freely over two pullies in the same horizontal line with the vertex at given distances on the same side of it. Determine the position of the cone's axis when in equilibrio.

---

19. If a straight lever, loaded at a given point with a given weight, be supported upon two props of unequal but known lengths. Determine the pressure sustained by each.

20. The ends of a lever are placed upon two

props of given lengths, but of unequal strength. Upon what point of the lever must a weight be placed, so that the pressure on each prop may be in proportion to its strength?

21. A cone of given weight rests upon two given props placed under the extremities of its axis; determine the pressure upon each.

22. A given weight is supported on two given props which stand at a known distance from each other on a horizontal plane, and which are prevented from sliding by means of a cord connecting their lower extremities. Determine the tension of the cord.

23. A given triangle without weight rests horizontally on three props placed at its angular points; determine a point in it, from which if a given weight be suspended the pressures on the three props may be in a given ratio to each other.

24. A given heavy triangle is supported in a horizontal position by three equal props, one of which is under the vertex, and the other two under the base. Determine the position of one of these, that of the other being given, so that the pressures on them may be equal.

25. Three unequal poles connected at their upper ends, and resting their lower on the ground in a triangle, support a weight. Compare the pressures on them in the directions of their length.

26. If *AE*, *BF*, and *CG*, be three levers, whose centres of motion are *E*, *F*, *G*; and which meet in a point *D*, from whence a given weight (*W*) is

suspended. Determine the pressures at their extremities.

27. Given the pressure upon one of the four legs of a rectangular table of known weight, find the pressures on the other three. And shew that without this *datum* the problem is indeterminate.

28. A table in the form of an equilateral and equiangular hexagon, of a given size and thickness, is supported by a point in the under surface, and weights of 7, 11, 15, 19, 23, and 27 pounds are suspended in successive order from the corners of the table. Determine the point of support.

29. A circular hoop is supported in a horizontal position, and three weights of 4, 5, and 6 pounds respectively are suspended over its circumference by three strings meeting in the centre. What must be their position, that they may sustain each other?

30. A given weight in the form of a triangle is supported by three known weights connected with the angular points A, B, C, by strings passing through a fixed ring at a given point D. Determine the lengths AD, BD, CD, and the angles they make with the vertical.

31. A given weight (W) is supported by (n) strings passing over pullies placed at the angles of a regular polygon whose plane is horizontal, each string being fastened to an equal weight (P). Determine the position in which (W) will rest.

32. A cylindrical bar is suspended by a given

point in a semicircle whose diameter is the bar. Find the inclination of the bar to the horizon, upon supposition that the semicircle is devoid of weight.

33. An inflexible lamina of metal in the form of a circular arc, of given weight, is placed upon a horizontal plane with two given weights (P) and (W) fixed at its extremities. Determine the position in which it will rest.—Also having given (P), determine (W), so that the arc may rest in a given position.

34. A spherical vessel of given weight and dimensions is loaded at a certain point of its edge with a given weight. Determine the position in which it will rest on a horizontal plane.

35. From what point must a thin wire in the form of a cycloid be suspended, so that when loaded at each end with half its own weight, it shall rest with its base perpendicular to the horizon?

36. (P) and (W) are two weights fixed to the ends of a given circular arc which is placed with its plane vertical on a plane inclined to the horizon at an angle (θ); shew that it will rest with the chord parallel to the plane when $P : W :: a - b.$ tang. $\theta : a + b.$ tang. θ; (a) being the sine and (b) the versed sine of half the arc.

---

37. Given a bent lever with arms of uniform thickness moveable in a vertical plane about the angular point: find the position in which it will rest.

38. A straight lever carries at one extremity a given weight, and to the other is attached a chain which reaches to the ground, and lies with part of its length loosely coiled up. Determine in what position the lever will rest.

39. If the arms of a lever be of different metals, whose specific gravities are as 2 : 3, and lengths as 1 : 2; determine the position in which it will rest.

40. If from the extremity A of a straight lever ACB, whose arms AC, CB, are equal, a catenary be described whose equation is $s^2 = 2ax + x^2$, and whose axis is CA produced; and from *any* point in the axis a weight be vertically suspended which presses upon the curve, shew that it will balance an equal weight hanging freely from B.

41. A weight slides on a thread fastened to the extremities of the equal arms of a lever of uniform density. Shew that the lever will not rest except in a vertical or a horizontal position; and that if it be put in motion, it will ultimately rest in a vertical position.

42. Having given a rectangular piece of metal of uniform thickness, it is required from one of its angles to draw a line cutting off a triangle, in such a manner that the remaining trapezium being suspended by the obtuse angle, the parallel sides of the trapezium may remain horizontal.

43. $DBCF$ being the half frustum of a right cone: determine a point in DB, by which if the solid be freely suspended, the slant side BC when quiescent may be perpendicular to the horizon;

DB, *FC*, and *DF*, being equal to R, r, and h, respectively.

44. If a triangle whose sides are in the ratio of 3, 4, 5, be suspended by the centre of the inscribed circle; shew that it cannot remain at rest unless the shorter side be in a horizontal position.

---

45. Determine the point in the curve surface, on which a semiparaboloid will rest on a horizontal plane.

46. A paraboloid laid upon a horizontal plane rests with its axis inclined to the horizon at an angle of 30°. Compare the length of the axis with the latus rectum.

47. Upon one side of a given straight line *AB* let a semicircle be described, and on the other an equilateral triangle *ADB*. If a solid be generated by there volution of this figure about *DC*, *C* being the centre of the semicircle, prove that it will rest upon the horizontal plane upon any point of its spherical surface.

48. A given hemisphere is placed with its convex surface on a given smooth inclined plane, and is kept from sliding down it by a string of given length, considered without weight, one end of which is fastened to a given point, and the other attached to the hemisphere. Determine the point where the hemisphere will touch the inclined plane when in equilibrio.

---

49. A beam, of given length and weight, is placed with one end on a vertical, and the other

on a horizontal plane. Determine the force necessary to keep it at rest, and the pressures on the two planes.

50. Determine the position of a straight rod, when it rests in equilibrio on a prop, and one end touching a vertical plane.

51. Place a straight rod loaded with any given weights, so that it may be in equilibrio, resting upon an upright prop, and one end touching a smooth vertical plane given in position.

52. A beam is supported by resting on a prop, and with one end on a curve surface. Determine the nature of the curve, so that the beam may be in equilibrio in any position whatever.

53. In the case of the preceding problem, if the nature of the curve be given; determine the position of the beam when it is in equilibrio.

54. A pole, of given length and weight, rests with one end on the ground against a wall, and the other attached to a string fixed to a given point in the wall. Determine the tension of the string.

55. A rod of given length and weight projecting obliquely from a wall, has its upper extremity sustained by two strings of given length, which are fixed to two given points in the wall. Determine the tension of the strings.

56. A beam, AB, rests with one end, A, on a prop. To the other is fixed a string, which passes over a pulley D, at the distance $AD=AB$, to which is hung a given weight. Determine in what position it will rest.

57. An uniform beam, *AB*, of given weight, is moveable round a hinge at *A*, and is kept in a given position by a weight (*P*), acting at the extremity of a string passing over a pulley at *C*, and attached to the beam at *B*. Determine the weight (*P*) when the points *A* and *C* are in the same vertical line.

58. A beam hangs, by means of a given cord fastened to its upper end, from a fixed point in a vertical wall. Against what point in the wall must its lower end be placed, that it may have no tendency either to ascend or descend? Within what limits for the length of the beam is this equilibrium possible?

59. A beam, of given weight, rests with one end on the ground, and with the other on an inclined plane; what is the force necessary to prevent the plane from moving?

60. A rod, of given weight and length, has a moveable weight attached to it, and is placed with one end against a vertical wall, and the other upon a horizontal plane. Determine the position of the moveable weight, when a given sustaining force is just sufficient to prevent the rod from sliding when in a given position.

61. A beam, of given length and weight, rests against a smooth vertical plane, and the other end is sustained by a given weight fastened to a string, which passes over a pulley placed at a given point. Determine the position of the beam when it rests in equilibrio.

62. A beam, of given length and weight, rests

with one end on a given inclined plane, and the other attached to a string passing over a pulley given in position. Knowing the weight P, affixed to the other end of the string, determine the position in which the beam rests.

63. A cylinder, of given weight and dimensions, is placed with its axis making an angle of 30° with the horizon; to the upper end of it is fixed a string, which passes over a pulley, and has a given weight attached to it. Determine the angle which the string makes with the side of the cylinder, to retain it in this position in a vertical plane: and what must be the direction to retain it in the same position, with the least weight possible. Determine also that least weight.

64. A given weight is sustained upon a cord, whose extremities are attached to rings, which move freely on two rods, inclined at given angles to the horizon, but in the same vertical plane. Determine the position of the weight when at rest, supposing it to have moved freely on the cord.

---

65. A given beam is supported on two given inclined planes. Determine the position of equilibrium.

66. If a straight, slender, uniformly dense rod, of given substance, be placed with its centre of gravity between a horizontal perfectly polished plane and the smooth edge of a vertical one; determine the tendency which its lower end has to slide along the horizontal plane.

67. If a given body ABC, be placed with its lower end A, on a smooth plane AD, making a given angle ADB, with the vertical plane BD, of given height, and resting upon the edge of BD, at B, which is smooth: and in this position be kept in equilibrio by a weight W, attached to a string passing over a pulley, the other end being fixed at C, and making a given angle with ABC; determine the weight W, and the pressure on each plane.

68. Let a slender heavy rod, AB, loaded with a weight, be placed with its upper end B, leaning upon an inclined plane DE, and its lower end A, upon a curve, situated in a vertical plane at right angles to the inclined plane. Determine the nature of the curve, so that the rod may rest in equilibrio in any position.

69. If a given beam be suspended by two strings, which pass over two fixed pullies in the same horizontal line, and a rod of given weight and length be fixed perpendicularly to the beam at its centre of gravity, having itself a ball of given weight fixed to its lower extremity. Determine the angle at which the rod will be inclined to the horizon, when two given unequal weights are attached to the extremities of the strings.

---

70. Two heavy spheres, of given dimensions, are placed in a hollow hemisphere, of given dimensions; determine their position of equilibrium.

71. Three spheres in contact, whose radii are r, r', r'', support another of given weight and magnitude. What weight does each supporting sphere sustain; and what are the horizontal pressures necessary to prevent the other from sliding?

72. If two perfectly polished spheres of given magnitude and weight, be put into a perfectly polished spherical vessel of given diameter; determine the distances of the points of contact, from the lowest point of the vessel, when the spheres rest in equilibrio.

---

73. Determine what must be the length of the frustum of a hexagonal pyramid, the sides of whose ends are R and r respectively; there being six points on the external surface, at a known distance from the less end, on which it will rest in equilibrio, on the point of a vertical prop.

74. A paraboloid rests upon a horizontal plane, with its axis vertical and vertex downwards. What must be the length of its axis, in order that the equilibrium may be that of indifference?

75. A hemispheroid resting upon its vertex; determine what must be the ratio between the major and minor axes, that it may just rest in a state of stable equilibrium.

76. A cylinder rests on a sphere with its axis vertical: determine under what limits of the base and altitude the equilibrium will be permanent.

77. If two hemispheres rest with the convex surface of one placed on that of the other; shew

that the equilibrium will be stable or unstable, according as the radius of the upper one is less or greater than three-fifths of the radius of the lower.

78. Determine the radius of the least sphere on which a given oblate spheroid can rest in a position of permanent equilibrium.

79. A paraboloid rests with its vertex upon that of a given hemisphere. Determine the length of its axis, so that it may all but fall.

80. A homogeneous elliptical spheroid rests on its smaller end, in a concave hemisphere; determine what the radius of the hemisphere must be, that the equilibrium may be stable.

81. A weight (W) is sustained on the circumference of a vertical circle by means of a string passing over a pulley at the highest point, from which (P) is suspended. Having given the ratio of $P : W$, determine the position of equilibrium; and also whether it is stable.

82. Determine the position of equilibrium of an uniform rod, one end of which rests against a plane perpendicular to the horizon, and the other on the interior surface of a given hemisphere.

83. A rod of given length rests with one end upon the concave surface of an inverted paraboloid, and passes over a point which is in its focus. Determine the position in which it rests.

84. An uniformly heavy thin rod, of given length and weight, is sustained in equilibrio, on the edge of a given hemispherical vessel, at some distance from its higher extremity, the lower

resting on the concave surface. Determine the position of the rod; and the pressures at the two points of support; the axis of the hemisphere being perpendicular to the horizon.

85. A thin conical stick being laid in a polished hemisphere; determine where it will rest in equilibrio, when both ends rest on the surface; and if made to slide, what curve will its centre of gravity describe?

86. If a body be balanced upon a horizontal plane, and a slight motion be given to it, its centre of gravity will move horizontally. Prove this, and shew in what cases the equilibrium is stable.

87. An inflexible rod, of given length, having a given weight attached, at a given distance from one of its extremities, is placed in a perfectly smooth hemispherical vessel, whose axis is perpendicular to the horizon. Determine the position in which the rod will rest, and the pressure at each extremity, the diameter of the hemisphere being known, and greater than the length of the rod.

88. APC, AWB, are two equal vertical quadrants, connected at A, where there is a pulley, over which passes a string, connecting two weights, (P) and (W). The string AP, passes along the curve, through a groove, whilst (W) hangs freely:—Having given the position of (P); determine that of (W), when there is an equilibrium.

89. Two equal weights, connected by a string, keep each other in equilibrio on a semicircle,

situated in a vertical plane, with its diameter horizontal: prove that the vertical pressure on any arc, beginning at the lowest point of the semicircle, varies as the versed sine of that arc.

90. A rigid prismatic bar, of uniform density and given length, is placed in the straight line joining two centres of force, whose distance is given, and whose intensities are in the ratio of 2 : 1. Determine the position of the bar, so that it may rest in equilibrio, $F \propto \frac{1}{D}$.

91. If particles of a spherical shell attract with forces varying inversely as the square of the distance, and a cylindrical rod of uniform density, whose length is (n) times the radius of the sphere, pass through the shell; determine the pressure on the shell when the rod is at rest; the part of it within the shell being equal to the radius of the sphere.

SECTION VI.

1. Let two equal forces in any directions AP, AQ, sustain each other on a tack A; find the pressure upon A.—Determine also when the pressure vanishes, and when it is a maximum.

2. Two equal weights are suspended by a string passing over three tacks which form an Isosceles triangle, the base being parallel to the horizon, and the vertical angle 120°. Compare the respective pressures on the tacks with each other and with the weights.

3. Prove that, in the case of the above problem, and also whatever be the vertical angle, the whole pressure, when estimated in the direction in which the weights act, is equal to the sum of the weights.

4. Let a cord $ACDB$ fastened at A and B support the weights (P) and (W) hanging from C and D. If AC and BD be produced till they meet the directions of the weights in d and c; when there is an equilibrium, the position of the cord will be such that $P : W :: Dd : Cc$. Required proof.

5. If a picture frame be hung up by only one string which passes through two rings at the upper angles A and B, and over two tacks C and D in the same vertical line CDE; prove that the

tension of the string is to half the weight of the picture :: $1 : \frac{CE}{CA} + \frac{DE}{DA}$; E being the point where CD meets AB.

6. In the preceding problem, if the angle at the lower tack be 120°, determine the place of the other tack, so that the tension may be one-third of the whole weight.

7. A string fastened at A, and passing over a fixed pulley B, has a known weight (W) hung by a knot at C, a point between A and B. Determine what weight must be appended at the end of the string, that CB may be horizontal.

8. A beam PQ, of uniform density and thickness, hangs by two strings AP, BQ, from two fixed points A, B. Shew that when there is an equilibrium, the tensions of the strings are inversely as the sines of the angles at P and Q.

9. A string having its extremities fixed to the end of a cylindrical beam of uniform density and given weight, passes over four tacks, so as to form with the beam a regular hexagon; determine the tension of the string, and the vertical pressure on each tack, the beam being horizontal.

10. Two equal weights are attached to a string passing over a cycloid whose base is horizontal. Determine the whole pressure; and prove that the pressure estimated in a vertical direction is uniform.

11. If two given equal weights sustain each other by a string passing over a smooth curve, the plane of which is vertical, the sum of the pres-

sures on any arc depends only on the directions of its extremities.

12. A chain of uniform density is suspended at its extremities by means of two tacks in the same horizontal line at a given distance from each other. Determine the length of the chain, so that the stress upon either tack may be equal to the chain's weight.

13. Supposing as in the last problem; determine the length of the chain so that the stress upon the tacks may be a minimum.

14. A cord, the ends of which are joined, is suspended freely over two pegs in the same horizontal line, so as to form two catenaries, of which the arcs are $(2s)$ and $(2s')$, and the tensions at the lowest points (a) and (a'). Prove that

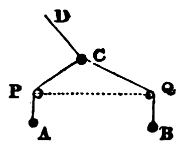
$$s - s' : a' - a :: a' + a : s + s'.$$

15. When a chain fixed at two points is acted upon by a central attractive or repulsive force, the tension at any point is inversely as the perpendicular let fall from the centre of force on the tangent at that point. Required proof.

16. If a string be fastened at two given points, and a heavy ring slides thereon; determine the length of the string so that the ring may rest at a given distance from one end.

17. A weight (C) is supported upon a string passing over two pullies not in the same horizontal line, and bearing two weights (A) and (B) fixed at its extremities. Having given the weights of (A), (B), and (C), determine by construction where (C) will rest.

18. If the ends of a thread of given length be fixed at two points in the same horizontal line, and at a given distance from each other; determine where two given unequal weights must be attached to the thread so as to remain in equilibrio in a line parallel and at a given distance from that joining the points of suspension.



19. A given weight (C) suspended from a given point D, is acted upon by two other given weights (A) and (B), by means of two strings passing over the pullies P and Q placed in given positions in the horizontal line PQ. Determine the position of each weight when there is an equilibrium.

20. Over two small pullies fixed at two given points in a vertical plane, a string is placed; one end of which bearing a weight is put through a ring fixed at the other extremity. Determine the position of the string when there is an equilibrium; the string and pullies being void of friction.

21. One end of a string of given length is fastened to an immoveable tack, whilst the other is held in some point of a line passing through the tack, and making an angle of 45° with the horizon, so that a heavy ring loose upon the string may rest in equilibrio. Determine the nature of the curve which is the locus of the ring.

22. One end of a perfectly flexible chain of given length and weight is fastened to a given point, the other passing over a pulley placed in

a given position. Determine what portion of the chain must pass over the pulley, so that the whole may rest in equilibrio.

23. If the ends of a perfectly flexible but inextensible string of given length be fastened at two given points on the surface of a sphere, and a small weight move freely along the string. Determine the position of the body when it is at rest on the surface of the sphere.

24. If the ends of a string of given length be fastened to two given points on the surface of a cone, and a small weight be at liberty to slide freely along the string. Determine the position of the weight when it is at rest on the surface of the cone.

25. Let a heavy flexible chain of given length be fixed to one extremity of an inflexible rod of given length, and of the same weight as an equal length of the chain: and when thus connected let the rod and chain be suspended at their other extremities from two given points in the same horizontal line. Determine the angle which the rod makes with the horizon when the whole is in equilibrio.

26. Let an elastic string of given weight, having its ends connected so as to form a flexible ring, be thrown over a solid of revolution whose axis is vertical; and being acted upon by gravity, let it descend till it rests in a horizontal position. Find its position of equilibrium: and determine what must be the nature of the curve that it may rest at all altitudes.

SECTION VII.

1. Two non-elastic balls move in opposite directions, $A : B :: 1 : 4$, and the momentum before impact is to that after $:: 3 : 1$. Determine the ratio of the velocities after impact.

2. A non-elastic ball A moves with a velocity (10) and strikes another B at rest, whose quantity of matter is (9). If A retains a velocity (3) after impact, determine its quantity of matter, and B's momentum after impact.

3. In the impact of non-elastic bodies, if the body struck be at rest before impact; it gains half the momentum, and half the velocity of the other by impact, if they are equal: more than half the momentum and less than half the velocity when greater than the impinging body; and less than half the momentum and more than half the velocity when less.

4. In the direct impact of non-elastic bodies, the difference between the sums of the products of each body into the square of its velocity, before and after impact, is equal to the sum of the product of each body into the square of the velocity gained or lost.

5. Given the diameters of two non-elastic balls which move in opposite directions with velocities a and b; determine their solidities when their motion after impact is a minimum.

6. There are five non-elastic bodies whose weights are 1, 3, 5, 7, 9, pounds; the first impinges on the second at rest with a velocity of four feet *per* second; the second on the third, and so on. What is the velocity communicated to the last?

7. A row of non-elastic balls, whose magnitudes increase in geometrical progression, are placed at equal distances in a straight line, and a given velocity is communicated to the first. Determine the time elapsed before the $(n)^{th}$ is put in motion.

8. A certain velocity (a) is communicated to each of two perfectly hard bodies at the instant of their impinging upon each other. Prove that the common velocity after impact is equal to $a \pm$ what would have been the common velocity if (a) had not been communicated?

---

9. If an elastic ball A, whose diameter is (a) moving upon a horizontal plane with a given velocity (v), impinge directly on another elastic ball B of equal density, and whose diameter is $(2a)$, at rest; determine their velocity after collision.

10. A and B are two perfectly elastic balls, whose magnitudes are given, and A's velocity before and after impact is a and b. Determine the velocity of B before impact.

11. Determine the ratio of two perfectly elastic balls, A and B, so that A, by striking B, may lose $\frac{1}{n}^{th}$ part of its motion.

12. A and B are two perfectly elastic balls, which are as $3 : 1$; their velocities are equal; and they move in opposite directions. Determine their motions after impact.

13. What must be the ratio between two perfectly elastic bodies meeting with equal velocities, so that they may move on together after impact.

14. A and B are two perfectly elastic bodies, and the velocity communicated from A to B at rest, is to the velocity retained :: $7 : 1$. Determine the ratio of the bodies.

15. A perfectly elastic ball, weighing (8) pounds, is moved with a velocity of (20) feet *per* second, and meets another weighing (10) pounds. After impact, the second ball is found to move with exactly the same velocity that it did before, but in an opposite direction. Determine its velocity.

16. A and B are two perfectly elastic balls, and they move with velocities which are as $2 : 1$. Determine their ratio, 1st, when B gains a velocity (2); 2dly, when A loses a velocity (1); 3dly, when A is at rest after impact.

17. A and B are two perfectly elastic balls, and A impinges upon B at rest with a given velocity. Given the magnitude of B, determine that of A, so that its momentum after impact may be the greatest possible.

18. A perfectly elastic ball (2) moving with a velocity (9), communicated a velocity (2) to a body (8), by means of an intermediate body. Determine its magnitude.

19. *A*, *B*, and *C*, are three perfectly elastic bodies, and $C=3A$. Find *B*, so that the velocity communicated to *C* through *B*, may be to the velocity communicated immediately from *A* to *C* :: 16 : 15.

20. In a series of perfectly elastic balls, increasing in geometric progression; determine their number, so that the motion may be increased (n) times.

21. Determine the ratio of increase of a series of perfectly elastic balls in geometric progression, whose number is (n), supposing the first to strike the second at rest, the second to strike the third at rest, and so on; and the velocity of the first to be to that of the last as $p : 1$.

22. *A*, *B*, and *C*, being the weights of three perfectly elastic balls, in the order of their magnitudes, *A* strikes *B* at rest with a given velocity, and drives it against *C*; the distance between *B* and *C* being given, and the velocity of *A*, find where *A* will overtake *B* again.

23. A row of (n) perfectly elastic balls, *A*, *B*, *C*, &c. is placed in a right line. Required their ratio, so that the motion of *A*, after impact, may be equally divided among all the bodies.

24. The magnitudes of three perfectly elastic bodies are in harmonic progression. Prove that the momentum communicated to either of the extremes by the impact of the other, is equal to the momentum of the mean moving with the velocity of the impinging body before impact.

25. If *A*, *B*, *C*, be any three perfectly elastic

balls, and a velocity be communicated from A to B at rest, and thence to C; then if $D : C :: A : B$, the velocity of A will be to that of $C :: \frac{1}{4}(A+B+C+D) : A$.

---

26. In the direct impact of imperfectly elastic bodies, prove that the sum of the products of each body into the square of its velocity before impact, is greater than the sum of the products of each body into the square of its velocity after impact.

27. The elasticity of two bodies, A and B, is to perfect elasticity :: 2 : 3. Required the ratio of $A : B$, so that A impinging upon B at rest, may, after impact, remain at rest. Determine also the velocity communicated to B.

28. A and B are two balls, whose elasticity is (e), and A strikes B at rest. Prove that if B be infinitely greater than A, A's momentum before impact is to the momentum communicated to $B :: 1 : 1+e$.

29. Determine the elasticity of two bodies, A and B, and their proportion to each other, so that when A impinges upon B at rest, A may remain at rest after impact, and B move on with an $(n)^{\text{th}}$ part of A's velocity.

30. If the elasticities of two balls, A and B, moving with velocities m and n, are as $e : 1$ and $e' : 1$ respectively, perfect elasticity being considered as unity; compare the relative velocities of the balls before and after impact.

31. A and B are two balls of given elasticity.

Determine the magnitude of a third ball of the same kind, so that the velocity communicated from A to B, by the intervention of this ball, may be equal to that communicated immediately from A to B. And determine within what limits the problem is possible.

32. A set of five balls, imperfectly elastic, are in geometric progression, whose common ratio is 2. The force of elasticity is to the force of compression as 3 : 2. Compare the velocity of the first with that communicated to the last.

33. Determine the force of elasticity so that, in the case of direct impact, the sum of the products of each body into the cube (or n^{th} power) of its velocity, may be the same before and after impact.

34. Two bodies, A and B, whose elasticity is (e), moving in opposite directions, with velocities (a) and (b), impinge directly upon each other. Find the distance between them when (t'') from the moment of impact have elapsed.

35. A ball, whose elasticity is (e), falls from a given height on a hard plane, and rebounds till its whole motion is lost. Determine how high it will rise at the $(n)^{\text{th}}$ rebound; and the whole space passed over. Having given also the height to which it rises, after (n) rebounds, determine its elasticity.

36. In the oblique impact of an imperfectly elastic body upon a plane, shew that the cotangent of incidence is to the cotangent of reflection as the force of compression to the force of elasticity.

37. If the compressing force be to the force of elasticity :: 1 : e. Determine at what angle such a body must be incident on a non-elastic plane, that the angle between the directions before and after impact may be a right angle.

38. Two bodies (1) and (2), moving with velocities (1), (2), whose elasticity is to perfect elasticity as 1 : 2, impinge upon each other, making the angles of 30° and 90° respectively, with the plane touching them at the point of contact. Determine the directions in which they will move, and their velocities after impact.

39. If one of two given non-elastic balls, touching each other, and resting on a horizontal plane, be struck by a third given ball, moving with a given velocity upon that plane, in any given direction, oblique to the line passing through the centres of the two quiescent balls: determine the motions of the balls after the impact.

40. A non-elastic ball strikes at the same instant two non-elastic equal balls, similarly situated with respect to the direction of its motion. Determine its velocity after impact, and also the velocities of the other balls.

41. A non-elastic ball strikes at the same instant two non-elastic equal balls: determine at what angle it must strike them, so that they may recede from each other the greatest distance possible.

42. A non-elastic ball A, moves with an uniform velocity (v) along a given line EGC; when its centre arrives at G, it impinges on a ball B,

whose centre is in the line GBD, inclined at an angle θ to EGC. Shew that A's velocity after impact $= v \sqrt{(\sin.^2\theta + \left(\frac{A}{A+B}\right)^2 . \cos.^2\theta)}$.

43. A and B are two equal balls at rest. Required their position, so that if a perfectly elastic ball C impinge upon them in a direction perpendicular to and bisecting the line joining their centres, the relative velocities of A and B after impact may be the greatest possible.

44. The centres of two non-elastic balls, B and C, are situated in two lines, AB, AC, which form a right angle, and placed in such a manner, that when a third non-elastic ball moving in a direction EAD, inclined at an angle of 30^0 to AC, arrives at A, it impinges upon them both. The balls A, B, C, being in the ratio of the numbers 3, 2, 1, respectively, and A's velocity being given before impact, determine its velocity and direction after impact.

45. Determine, geometrically, the effects of the collisions when a body strikes any number of bodies at once, in any directions whatever, supposing the bodies to be non-elastic.

46. Two elastic balls, A and B, which are in the proportion of 3 : 1, are placed on a horizontal plane. A impinging with a given velocity on B at rest, drives it perpendicularly against a non-elastic vertical plane, and it meets A in returning at half its original distance. Determine the elasticity of the balls and the time of motion.

47. Determine the diameter of a perfectly elas-

tic ball, which moving with an uniform velocity, and striking a given ball B at rest, shall communicate a velocity to it, which shall cause it, after being reflected at a given hard vertical plane, to meet the striking ball in a given point.

48. Given the point A between two inclined planes: determine the direction of projection such, that the body, after reflection at each plane, may return to the hand.

49. If two equal non-elastic bodies move from the base of an Isosceles triangle, whose vertical angle is 60°; determine the motion after impact.

50. Determine the locus of a point C, between two perfectly elastic planes, from whence a perfectly elastic ball being projected, may impinge first upon one, then on the other, and return to the point C.

51. Between two parallel planes, perpendicular to the horizon, project a body from a given point, so as to return to the hand after (n) reflections.

52. If a non-elastic body move uniformly along one side of a regular polygon, shew that it will continue to describe the other sides uniformly, but with velocities decreasing in geometric progression. And in the case of a hexagon, shew that the time of describing the first side is to the time of describing the last as 1 : 32.

53. A and B are two given points in the diameter of a circle: determine in what direction a perfectly elastic body must be projected from A, so that, after reflection at the circle, it may strike B.

54. A non-elastic body moving uniformly along the arc AD, of a semicircle ADB, whose diameter is horizontal, impinges on a vertical plane DC; prove that it will move uniformly along DC, and that the time along DC will vary as the tang. BD.

55. An imperfectly elastic ball, being projected from P, a point in the periphery of a circle PAB, whose centre is C, after impinging at A and B, returns to P. Determine the value of the angle CPA.

56. A person standing in the centre of a given circle throws a perfectly elastic ball with a given velocity along a plane coinciding with the horizontal radius which is reflected to him by a small non-elastic plane coinciding with the tangent. Supposing the plane to revolve about the centre of the circle, determine in fixed space the locus of his position on the plane, when the ball projected with the same velocity and reflected by the tangent plane, always returns to his hand in the same time.

57. A and B are perfectly elastic bodies; and A moving in the circumference of a circle, strikes B at rest, and makes it move uniformly in the circumference. Determine where they will next meet.—Determine also the ratio of $A : B$ so that they may meet when A has moved over 180° from the point of impact.

58. A and B are two imperfectly elastic bodies; and A moving in the circumference of a circle, strikes B at rest, and makes it move uniformly in the circumference. Determine where

they will meet after $(2n)$, and $(2n+1)$ strokes; the space described; and the time elapsed. Shew also that after an infinite number of strokes the velocity of A will be the same as if the bodies were perfectly elastic, and of B as if non-elastic, and one impact only had taken place.

59. If $BCDA$ be a quadrilateral figure, whose side AB is the diameter of the circumscribing circle; and a perfectly elastic ball moving in the direction CD impinge upon the plane DA; shew that it will, after being reflected again by AB, return to C.

60. A ball having descended to the lowest point of a circle through an arc whose chord is C, drives an equal ball up an arc whose chord is c. Shew that the common elasticity (e) of the two balls may be found from the proportion

$$1 : e :: C : 2c - C.$$

61. If a perfectly elastic ball be struck from either focus of an ellipse in any direction, it will return after two reflections from the curve to the same point.

62. From what point in the periphery of an ellipse may an elastic body be so projected as to return to the same point after three successive reflections at the curve, having in its course described a parallelogram?

63. A given parabola is placed with its axis vertical, and vertex downwards: determine from what point of its highest ordinate an elastic body must be let fall, so that after impinging once on the curve, it may strike the vertex.

64. A given parabola is placed with its axis vertical, and vertex downwards: determine from what point of its highest ordinate an elastic ball must be let fall, so that impinging six times on the curve, it may return to the same point.

65. If the position of a ball be given on a triangular billiard table: determine the direction in which it must be struck, so that after rebounding from all the sides it may strike another ball whose position is also given. And shew that there are three directions, in any one of which if the ball be struck, it will pursue continually the same path after being twice reflected from each of the sides.

66. In what direction must a perfectly elastic ball impinge upon a side of a billiard table, that after a second reflection from an adjacent side of the table, it may strike another ball given in position?

67. In what direction must the ball be struck so that it may impinge first on the adjacent side of the table, then on the opposite side, and then on a ball given in position?

68. The sides AB, CD, of a billiard table are parallel, and an imperfectly elastic ball struck from a point C in one side impinges at E in the other, and is reflected to D in the first side. Shew that the time along CE is to the time along ED as the force of elasticity to the force of compression.

69. If two sides of a billiard table be inclined at an angle of 5°, and a perfectly elastic body be projected against one of them at an angle of 7°;

find after how many reflections it will cease to approach the angle.

70. A and B are two elastic balls placed on a billiard table, and FC the reflecting cushion. Join AB and produce it to C; then if A impinge on B, and drive it against the cushion, shew that the balls will meet after the reflection of B, if the angle of impact is equal to $45^{\circ} - \frac{1}{2} ACF$.

71. In what direction must a ball be struck from a given point at one *end* of a rectangular billiard table, so that after impinging upon the *sides* any number of times, it may strike a given point in the other *end*?

72. If the sides of a rectangular billiard table $ABCD$ be given: and a ball P be struck so as to impinge upon the three sides AB, BC, CD, in the points E, F, and G, and thence pass into the pocket at A; the distance of A from G being equal to AB one of the longer sides of the table. Determine the least distance of P from A.

73. $ABCDE$ is a pentagonal billiard table with unequal, but given, sides and angles. It is required to find that point in one of its sides, and the direction of impact such that an elastic ball may continually describe the same path, striking every side of the table in succession.

74. Determine the direction in which a perfectly elastic billiard ball must be struck, so that moving from a given point on a five-sided billiard table, it may, after impinging on the first, third, fifth, second, and fourth sides in order, be reflected to a given point.

75. Two perfectly elastic balls A and B are placed in given positions upon a table in the form of an irregular hexagon. Determine the direction in which A must be struck, so that after impinging upon each of the sides, it may strike B.

76. Given the position of two perfectly elastic balls on a table whose sides form a given polygon; determine the direction in which one of the balls must be struck, so that after impinging upon each of the sides in succession it may strike the other ball.

77. A ball half elastic having fallen from a given point, at the middle of its descent is projected parallel to the horizon with the velocity acquired. Determine the position of a non-elastic plane, which, opposed perpendicularly to its motion, will cause it to move to the same point to which it would have descended in the right line; and find the whole time of its motion.

78. An elastic sphere, after impinging on a non-elastic plane, (not smooth,) is observed to move off at an angle equal to the angle of incidence. Determine the sphere's elasticity.

79. A weight (w) is connected by a string, passing over a fixed pulley, with a number of equal balls (p) placed in a vertical cylinder whose diameter is equal to that of the balls. They are connected to each other by elastic threads equal and given in length. Determine the velocity of (w) when (n) balls are drawn up, and the number required to destroy (w)'s velocity.

80. An imperfectly elastic body revolving in an

ellipse whose eccentricity is $(\frac{1}{2})$, is reflected at the mean distance by a plane coincident with the distance, so as to move after impact in the direction of the axis minor. Determine the degree of elasticity; and compare the periodic time in the two ellipses, $F \propto \frac{1}{D^2}$.

81. A perfectly elastic body is projected due North from the equator at an angle of 45°, and with a velocity equal to the velocity in a circle at that point; and when it reaches the earth again, it impinges on a non-elastic plane which is parallel to the horizon at that point. Determine the path of the body.

82. If a perfectly elastic body at a given distance *CP*, fall from rest towards a centre of force *C*, the centripetal force varying as the distance from the centre directly, determine where a plane inclined at an angle of 45° to *PC* must be fixed, so that the body after reflection may describe an ellipse whose axes shall be in a given ratio.

83. A perfectly elastic body begins to fall from a given distance in a straight line towards the centre of force *S*; the force varying inversely as the square of the distance: in its descent it impinges upon a non-elastic plane inclined at a given angle to the direction in which it is falling, and after describing a certain curve comes to the plane on the other side, from whence it is reflected into the centre. Determine the nature of the curve, and compare the whole time of the body's motion with the periodic time in a circle

whose radius is equal to the distance from which the body began to fall.

84. At equal distances on each side of a centre of force S, are placed two inclined planes making an angle of 45° with a line passing through the centre; the law of the force being the inverse square of the distance. If a perfectly elastic body begins to descend down this line from a given altitude above the upper plane, and is reflected, determine its subsequent motion.

85. AB and CB are two equal lines at right angles to each other, and between them is placed a line DE equal to either of them. Two equal and perfectly elastic balls fall from A and C towards a centre of force at B, the force varying directly as the distance, but at E and D are reflected by planes inclined at angles of 45° to AB and CB. What will be the subsequent motions of the balls?

86. The earth being supposed spherical, a perfectly elastic ball is projected in a direction making an angle of 45° with the horizon, and with a velocity equal to the velocity in a circle at the same distance; shew that after three rebounds it will return to the same point; and the whole time of motion will be to the periodic time in a circle at the earth's surface :: $2. (\pi+\sqrt{2}) : 2$.

87. In the preceding problem, find the position of a circular ring, so that the ball may just touch it; and supposing a body to fall from this ring, determine the whole time of descent to the earth's centre.

88. The axes of an ellipse, in one focus of which is a fixed centre of force, are in the ratio of $\sqrt{2} : 1$. A perfectly elastic body descending from a distance equal to the axis major, in the direction of the radius vector passing through the extremity of the axis minor, impinges on the ellipse. Determine what will be its subsequent motion; the law of the force being the inverse square of the distance.

89. If through the earth, supposed to be a perfectly homogeneous sphere of given radius, a hole be perforated diametrically, and three non-elastic balls, whose quantities of matter are 1, 2, 3, respectively, be let fall at the interval of (n) seconds from each other; determine when and where they will come together, the perforation being supposed a vacuum.

SECTION VIII.

1. Determine the time of a heavy body's falling through 100 feet, and also the velocity acquired.

2. Through what space must a heavy body fall to acquire a velocity which will carry it through 144 feet in one second?

3. A heavy body falls down one-third of the vertical altitude in the last second of time. Determine the altitude and the whole time of descent.

4. A heavy body falls through the last $(m)^{th}$ part of a tower in (n) seconds. Determine the height.

5. If sound be supposed to move uniformly through 1142 feet *per* second; determine the depth of a well when the time of a heavy body's falling to the bottom is equal to the time of the sound's rising.

6. A heavy body in the third second of its fall descends through a space which is to the space described in the last second but three as 5 : 4. Determine the whole time of its fall.

7. A heavy body falls from a tower 200 feet high. Determine the time of its falling through a part whose length is two-thirds of its height, and which is so situated that its extremities are equidistant respectively from the top and bottom of the tower.

8. A stone being let fall into a well, it was observed that (n) seconds elapsed between the time of its delivery and the time when the sound of the fall at the bottom reached the ear. Determine the depth of the well.

9. At one extremity A, of a given horizontal line AB, a cannon is discharged: determine how high a perpendicular, BC, must be raised, so that a heavy body falling down it may reach the ground at B, at the same time that the sound arrives at C, but not till $\frac{4}{5}$ of a second after the sound arrives at B.

---

10. A heavy body comes to the ground from the top of a tower in $2''$; determine the velocity of projection.

11. If a body be projected perpendicularly downwards, with a velocity of (a) feet *per* second: determine the space described in (n) seconds.

12. A body is projected perpendicularly upwards, with a velocity of 64 feet *per* second; determine the time of ascent through 63 feet.

13. To what height will a heavy body rise in (n) seconds, if projected perpendicularly upwards, with a velocity of (a) feet *per* second.

14. A body is acted upon by a force which is equal to $\frac{7}{8}$ of that of gravity for $\frac{8}{7}$ of a second. Determine the space described, and the velocity acquired.

15. A body is acted upon by a force which is

to the force of gravity as 1 : 4, and moves through 360 feet: for what time does it move; and what velocity will it acquire?

16. A heavy body, projected perpendicularly downwards with a velocity of (a) feet per second, descended in the last second of its fall, a space which was to the space descended in the second immediately preceding as $m : n$. Determine the time of descent, and the height from which the body was projected.

17. A heavy body projected perpendicularly upwards from the bottom of a tower, with a velocity acquired through $\frac{2}{5}$ of its height, rose to the top in 2″. Determine the height of the tower.

18. How far will a heavy body descend in 12 seconds, when acted on by a constant force, which is to the force of gravity as 7 : 4?

19. If two bodies, acted upon by constant moving forces, in the proportion of 5 : 4, describe spaces from rest in the proportion of 4 : 5, and acquire velocities in the proportion of 5 : 6. Determine the ratio of their quantities of matter.

20. If a body be moved from a state of rest by the action of an uniform force, the space described at the end of any time, is equal to that which would be described in the same time with the mean velocity continued uniform.

21. Shew that a body cannot move so that the velocity shall vary as the space from the beginning of the motion. And if the velocity vary as the cube root of the space, determine how the time and the force vary.

22. A bow is drawn by a force of 50 pounds, the weight of the arrow being $\frac{1}{16}$ of a pound. Compare the force of gravity with the initial accelerating force which the string exerts upon the arrow, when it is let go; neglecting the inertia of the bow.

23. If a cylinder be placed with its axis horizontal, determine the greatest distance to which it may be produced, so that a bullet fired from the one end, with a given velocity, may just pass through it.

24. A musket-ball, of given diameter, discharged at a target of given substance and thickness (at a distance not affecting its initial velocity) with a velocity of (a) feet *per* second, passed through the target, and struck a bank of earth at the distance of (b) feet. Determine the depth the ball will penetrate into the bank.

25. A ball, weighing 24 pounds, strikes a wall with a velocity of 1700 feet. Determine the weight of a beam terminated by a hemisphere of the same diameter as that of the ball, which, when moved with a velocity of 10 feet, may penetrate to the same depth; and the weight of a similar beam, which may have the same effect in *shaking* the wall.

26. Suppose a spring to be compressed through a space of 6 inches, and that a weight or pressure of 64 pounds retains it in that position: then if the spring be loosed, and suffered to act on a weight of 5 pounds, determine the velocity generated in that weight, with the time of accelera-

tion for any space: the force of the spring being exerted horizontally.

27. If a weight of 80 pounds, with a velocity of 32 feet *per* second, striking against a spring, be found to bend it two inches: determine the elastic force of the spring so bent, with the velocity lost, and the time of describing each eighth part of an inch.

28. If a body fall by the action of an uniform force, and describe (a) and (b) feet in the (m)th and (n)th second respectively, reckoning from the beginning of the motion; determine the space described in the (x)th second.

---

29. A heavy body is let fall from a given height; but after it has fallen through (a) feet, another is projected from the same point, with such a velocity, that the two bodies come to the ground at the same time. Determine the velocity of projection.

30. From what height must a ball descend freely by the force of gravity, when the square of the height it descends through in the last second of time, multiplied by the cube of the height it had descended through before the last second, is a maximum.

31. A heavy body is let fall from one end of a vertical line at the same time that another is projected upwards from the other end, with the velocity which would be acquired by falling down the line. Determine where they will meet.

32. Given the velocity (a) with which a heavy body is projected downwards, and the velocity (b) with which another is projected upwards at the same time; determine where they will meet.

33. A heavy body is projected upwards from the lower extremity of a given vertical line with a given velocity. After what time must another be projected downwards from the upper extremity, with the same velocity, so as to meet the former in the middle point of the line.

34. If two heavy bodies are moved at the same time towards each other, from the two extremities of a vertical line (L); one projected upwards with a velocity $\left(\frac{3L}{2}\right)$, the other let fall from rest: determine where they will meet.

35. A heavy body has fallen from A to B, when another body is let fall from C, a lower point in the same vertical line; how far will the latter body fall before it is overtaken by the former?

36. A body is projected from the bottom of a vertical line, with a velocity which will carry it up to B; at the same time, a body is let fall from A, a given point below B, in the same vertical line; where, and in what time, will they meet?

37. From two ends of a vertical line two bodies are, at the same instant, projected towards each other, with velocities (v) and (v'), determine their distance from each other when $(\frac{1}{n})^{th}$ part of the time in which they would meet, has elapsed.

38. If at the same time that one heavy body begins to descend, another begins to ascend with

such a velocity as it might acquire by falling through half the whole space; determine where they will meet. And shew what difference would be occasioned in the time of meeting, if the latter body was projected with half the velocity acquired through the whole space.

39. A heavy body is projected downwards from the top of a tower, with a velocity acquired down half its height; and at the same time another is projected upwards from its base, with the velocity acquired down twice its height; and these meet in $2''$. Determine the height of the tower.

40. A heavy body falling meets another that was projected upwards from a certain point with a velocity that the falling body would have acquired at that point; but when they meet, the falling body has been in motion (n) times longer than the rising body. Determine the ratio of the spaces described.

41. If two heavy bodies be projected towards each other at the same time, from the two ends of a vertical line, with velocities (v) and (v'), they will always meet in the middle of the line, if the difference of the squares of the velocities, (v) and (v'), be equal to the square of a velocity acquired in falling down half the line.

42. With what velocity must a heavy body be projected downwards, that in (n'') it may overtake another body, which has already fallen (a) feet?

43. If a heavy body, projected upwards, meet another let fall from the top of a vertical line, at the distance of $(\frac{1}{m})^{th}$ of the line from the vertex.

Determine the space fallen through to acquire the velocity of projection.

44. A person ascending in a balloon, lets fall a stone when at a given height. Determine the time (t) of the stone's reaching the ground, supposing the velocity of the balloon at the given altitude known :—and explain the meaning of the negative value of (t).

45. Two bodies are projected at the same time with velocities (v) and (v'), from the two extremities of a vertical line. Prove, *geometrically*, that if they meet in the middle point of the line, $v \backsim v'$ is equal to the velocity acquired in the time of meeting.

46. Two bodies, A and B, move in opposite directions with velocities, the sum of which is given. Shew that the sum of the products of each body into the square of its velocity is a minimum, when the velocities are reciprocally proportional to the quantities of matter in the bodies.

47. Two equal non-elastic bodies are projected at the same instant towards each other, from the two extremities of a vertical line, each with the velocity which would be acquired in falling down it. Determine the time which elapses between their impact and their arrival at the lower extremity of the line.

48. From what height above A, must a heavy elastic body be let fall, so that it shall return to A, after being reflected at B, in the least time possible?

49. An imperfectly elastic ball is projected with

a given velocity against a non-elastic horizontal plane, and being reflected just reaches the point of projection in (t''). Determine the distance of the point of projection from the plane; and the elasticity of the body.

50. Two equal and elastic balls are let fall at the same instant in the same vertical line from two altitudes ($9a$) and ($4a$) above a horizontal plane. Determine the successive points of impact, and the spaces described by each before they return to their original positions.

51. A perfectly elastic ball falls from a point A to the horizontal plane at C, and is reflected back to a point B below A, in (n''); having given $BC = (a)$ feet, determine the distance AB. Find also AB, when (n) is a minimum.

52. The points A, B, C, &c. are taken equidistant from each other in the vertical line AS. A body half elastic, beginning its descent from A, impinges on a non-elastic horizontal plane at B, and in its second descent against another at C, and so on. Find the space described between the $(n-1)^{\text{th}}$ and $(n)^{\text{th}}$ rebounds and the time of describing it.

53. Two imperfectly elastic bodies A and B are at a given distance in the same vertical line: A the higher is acted upon by gravity, which is supposed to have no effect on B. Shew that if A fall and strike B successively, the intervals between the strokes decrease in geometric progression; and determine the space passed over after any number of strokes.

54. A perfectly elastic ball A falls from the upper extremity of a given vertical line AB, and at the same time another perfectly elastic ball B is projected upwards from a horizontal non-elastic plane at the bottom; they meet in some point C; and are reflected back. Determine the point C, so that they may ascend and descend from it continually. And find the velocity of B at that point.

55. An imperfectly elastic body descending vertically from rest meets a horizontal plane which is moving uniformly in an opposite direction. Having given the distance between the plane and the body at first, and the degree of elasticity; determine what must be the velocity of the plane, so that the body may return to the point from which it fell.

56. Two elastic balls A and B are connected by a small thread PQ, B being the lower. If A be let fall so that both begin to descend together by the sole force of gravity in the same vertical line, B being reflected at a horizontal plane FG will meet A in its descent. Having given PQ, determine the height from which A must fall so that B may meet it at a given distance from the horizontal plane.

57. From what height must a body descend towards a given horizontal plane AB, so that the time of falling through CB and describing BA with the acquired velocity may be a minimum.

58. If two balls A and B begin to fall at the same time from two points in the same vertical line, and with the velocities acquired are inflected

along the horizontal plane till one overtakes the other. Shew that the time of A's descent is equal to the time of B's uniform motion.

59. Two elastic balls beginning their descent from different points in the same vertical line, are inflected along the horizontal plane with the velocities acquired, by a non-elastic plane inclined to their direction at an angle of 45^0. Given the distance at which one impinges upon the other on the horizontal plane, and the point from which one of them descended, to find the point from which the other began its motion.

60. In the case of the preceding problem, shew that if a circle be described passing through the two points from which the balls began their motion, and touching the horizontal plane, the point of contact will bisect the distance between the vertical line and the point where they impinge on each other.

61. On a given straight line perpendicular to the horizon, as diameter, describe a circle, and draw a tangent at its higher extremity. Then if from any point in this tangent a straight line be drawn parallel to the diameter, cutting the circle in two points, from which horizontal chords are drawn; the times of falling down the parts of this line, together with the times of describing the corresponding chords with the velocities acquired at their extremities, are equal.

62. If a body be projected obliquely upwards, shew that the square of its velocity will always be equal to the square of the velocity of projection,

diminished by the square of the velocity which it would acquire by falling down its perpendicular height above the horizontal plane passing through the point of projection.

63. If a body fall (n) feet the last second; determine the height fallen from, supposing the force of gravity to vary inversely as the square of the distance.

64. Compare the space described in 1″ by the force of gravity in any given latitude, with that which would be described in the same time if the earth did not revolve round its axis.

65. Let two bodies move with known uniform velocities in given directions AP, BQ: determine when their distance will be a minimum.

66. Two bodies A and B move from the opposite extremities of the diameter of a semicircle, A along the curve, and B along the diameter, and they reach the farther extremities of the diameter at the same instant. Determine the diameter of the semicircle, the whole time the bodies are in motion, the time of their nearest approach, and their distance and position at that time, A moving with a velocity of (a) feet the first second, ($a+b$) the second, ($a+2b$) the third, and so on, and B with an uniform velocity of (c) feet *per* second.

---

67. From what height above an inclined plane of 30° inclination must a body (P) be let fall, so that its effect may be equal to that of a weight (mP) falling through (n) feet on an inclined plane whose elevation is 45°?

68. A body falls 9 feet along an inclined plane in one second. Determine the inclination of the plane.

69. If the elevation of a plane be 30°, how far will a body run down in one second?

70. A body falls from rest by the force of gravity down a given inclined plane: compare the times of describing the first and last halves of it. Compare also the time of falling down $\frac{1}{n}^{\text{th}}$ part with the time of falling down the remainder.

71. Given the base of an inclined plane: determine its height, so that the time of descending down the plane may be a minimum.

72. A heavy body in falling down the length of an inclined plane, which was (a) feet, describes (b) feet in the last second of its fall. Determine the plane's height.

73. If a heavy body fall down a given inclined plane; compare the space described in the first (n) seconds with that described in the last (n) seconds.

74. Divide the length of a given inclined plane into three parts, so that the times of descent down them may be equal.

75. Determine the time of describing 30 feet on a plane inclined to the horizon at an angle of 30°, the force of gravity being supposed to be diminished by one-fourth of its present quantity.

76. The times of falling down the length and height of an inclined plane being in the ratio of $m : n$; determine the plane.

77. If two heavy bodies begin to fall at the

same time from the common vertex of two inclined planes, the line joining them will move parallel to itself.

78. There are two inclined planes whose common altitude is (a) feet, and lengths are such that a heavy body is (t) seconds longer in falling down one than down the other; and two bodies will support each other in equilibrio, when they are as $m : n$. Determine the lengths of the planes.

79. Determine the velocity with which a body must be projected from the top of an inclined plane, so as to descend down the length in the same time that it would fall freely down the height.

80. If two heavy bodies be projected with equal velocities from the same point, the one directly downwards, the other along an inclined plane: compare the spaces described by them in any given time.

81. Two bodies projected along two planes inclined to the horizon at angles of 45° and 30° respectively describe spaces as $\sqrt{2} : \sqrt{3}$. Required the ratio of their initial velocities.

82. A body is projected down an inclined plane with the velocity acquired in falling down its height, and it describes the length of the plane in the time of falling down its height. Determine the elevation of the plane.

83. A heavy body is let fall from the top of an inclined plane down the length, and at the same time another is projected from the bottom with the velocity acquired through the whole length:

determine where they will meet. Determine also the point of meeting, when the body is projected with the velocity acquired through (n) times the length, and compare this latter distance with that which would be described if the body were projected with (n) times the velocity acquired down the length.

84. Determine the proportion of a plane's height to its length, so that the time of describing the length may be equal to the time of falling freely down the height together with the time of describing the base with the last acquired velocity continued uniform.

85. A body falls down a given length of an inclined plane, and impinging upon the horizontal plane moves along it. Determine the elevation of the plane, when the time of moving upon the horizontal plane over a space equal to the height of the plane is equal to the time down the plane.

86. A body falls down an inclined plane, and is made to move along the base which is given, with the velocity acquired. Determine the height of the plane, so that the whole time may be a minimum.

87. A body is projected down the length of a given inclined plane with a velocity (v), and at the same time another is projected from the bottom with a velocity (v'); determine where they will meet.

88. A body is projected from the top of a given inclined plane with a velocity (v), and after it has moved (n'') another is projected from the

bottom with a velocity (v'); determine where they will meet.

89. A body projected upwards along an inclined plane with a velocity of (n) inches in 1″, describes (r) inches before it stops. Determine the ratio of the plane's height to its length.

90. A heavy body is projected up an inclined plane whose length is (n) times its height, with a velocity of (a) feet *per* second. Determine in what time its velocity will be destroyed.

91. From the vertex of a conical spire whose slant side is four times the length of the diameter of the base, two heavy bodies were let fall at the same time, the one down the axis and the other down the slant side: and it was observed that the sound of the body which fell down the axis, reached the extremity of the slant side just at the same instant as the other body. Determine the height of the spire.

92. If a tube be wound round a tower at an angle of 60°, and a body fall through it in (n) seconds: determine the height of the tower.

93. If a spiral tube wind round the surface of a paraboloid standing on a horizontal plane, and make a given angle with the generating parabola; determine the accelerating force on a body descending in the tube; and prove that if it descends from a point very near the vertex, it will make its successive revolutions in equal times.

94. Given the length of an inclined plane $=2a$, and its altitude $=a$; and a heavy body descends down the plane in the same time that another

moves uniformly along the base. To determine their nearest approach.

95. From a point in the intersection of two planes; inclined to each other at a given angle, two balls were let go, and at the time when they were (a) feet asunder, they were in the same vertical line. Determine the inclination of each plane to the horizon, and the time of descent of each ball.

96. AB and AC are two lines inclined to each other at a given angle, the former indefinite, and the other of a given length. A body (P) begins to move from A, along AB, at the same instant that another body (P') begins to move from C, along CA; the former moving with an accelerated velocity of 1 foot in the first second, 5 in the next, 9 in the third, and so on: and the latter body with an uniform velocity of 6 feet in a second. Determine the time of their nearest approach to each other; and their position and distance at that time.

97. Two planes, equal in length, are inclined at angles of 45° and 30° respectively to the horizon. A body is projected downwards from the top of the first, and another upwards from the bottom of the second, each with the velocity acquired down a vertical line, equal in length to either plane. Compare the times of describing each plane, and the velocities at the end of the motions.

98. If from two given points on the same inclined plane, two heavy bodies are suffered to descend along two given straight lines, at the

same time from the common vertex of two inclined planes, the line joining them will move parallel to itself.

78. There are two inclined planes whose common altitude is (a) feet, and lengths are such that a heavy body is (t) seconds longer in falling down one than down the other; and two bodies will support each other in equilibrio, when they are as $m : n$. Determine the lengths of the planes.

79. Determine the velocity with which a body must be projected from the top of an inclined plane, so as to descend down the length in the same time that it would fall freely down the height.

80. If two heavy bodies be projected with equal velocities from the same point, the one directly downwards, the other along an inclined plane: compare the spaces described by them in any given time.

81. Two bodies projected along two planes inclined to the horizon at angles of 45° and 30° respectively describe spaces as $\sqrt{2} : \sqrt{3}$. Required the ratio of their initial velocities.

82. A body is projected down an inclined plane with the velocity acquired in falling down its height, and it describes the length of the plane in the time of falling down its height. Determine the elevation of the plane.

83. A heavy body is let fall from the top of an inclined plane down the length, and at the same time another is projected from the bottom with the velocity acquired through the whole length:

determine where they will meet. Determine also the point of meeting, when the body is projected with the velocity acquired through (n) times the length, and compare this latter distance with that which would be described if the body were projected with (n) times the velocity acquired down the length.

84. Determine the proportion of a plane's height to its length, so that the time of describing the length may be equal to the time of falling freely down the height together with the time of describing the base with the last acquired velocity continued uniform.

85. A body falls down a given length of an inclined plane, and impinging upon the horizontal plane moves along it. Determine the elevation of the plane, when the time of moving upon the horizontal plane over a space equal to the height of the plane is equal to the time down the plane.

86. A body falls down an inclined plane, and is made to move along the base which is given, with the velocity acquired. Determine the height of the plane, so that the whole time may be a minimum.

87. A body is projected down the length of a given inclined plane with a velocity (v), and at the same time another is projected from the bottom with a velocity (v'); determine where they will meet.

88. A body is projected from the top of a given inclined plane with a velocity (v), and after it has moved (n'') another is projected from the

regard to the vertical, so that the rectangle under their bases may be the least possible.

109. The friction of a body being supposed independent of velocity; determine an expression for the time of a body's descent down a given inclined plane, the friction being equal to $(\frac{1}{n})^{th}$ part of the pressure.

---

110. Through what chord of a circle must a heavy body fall to acquire half the velocity it would gain in falling down the diameter?

111. There are two circles, whose radii are as 2 : 1; compare the time of descent down a chord of 90°, in the former, with that down a chord of 60° in the latter.

112. Determine the ratio of the velocity acquired down a chord of 60°, and that acquired down a chord of 120°.

113. If two circles, whose diameters are in the same vertical line, touch each other internally, and any chords be drawn from the point of contact; compare the times of falling through the portions intercepted between the circumferences, the motion beginning at the point of contact.

114. If two circles, whose diameters are in the same vertical line, touch each other internally, shew that the times down any chords of the lesser circle are equal, the body being supposed always to begin its motion from the extremity of the corresponding chord of the greater circle.

115. In a given circle, the plane of which is

vertical, to draw a diameter which shall be described in any given time, not less than the time in which the vertical diameter is described.

116. Compare the time of a body's falling down the vertical diameter of a circle, with the time of its descent down two chords, drawn from its extremities to a given point in the circumference, no velocity being lost in passing from one chord to the other.

117. Find a point in the circumference of a vertical circle to which a body may fall down an inclined plane from the centre in the same time that another would fall down the diameter.

118. A body in falling down a plane AB, in the time (t), acquired a velocity (v); determine the plane down which it must fall to acquire the velocity (v'), in the same time.

119. A body in falling down a plane AB, in the time (t), acquired a velocity (v). Determine the plane down which it must fall to acquire the velocity (v') in the time (t').

120. Prove from a property of the sphere, that the time down any chord of a circle, whose plane is inclined to the horizon, is equal to the time down the diameter.

121. AB is a vertical line of given length. Determine the locus of a point P, such that the square of the time down AP, together with the square of the time down PB, may be always the same.

122. Determine the position of a straight line, down which the time of falling will be equal to

twice the time down the same line, when perpendicular to the horizon.

123. If A be the lowest point of a circular arc AB, and AI be taken equal to $\frac{1}{2}AB$, and the chord AO be taken to the chord AI, in the ratio of $\sqrt{2}:1$; prove that the time down BO is equal to the time down the remaining arc OA.

124. The axis of a parabola being vertical, shew that the time of descending down any chord drawn to the vertex, is greater than the time of falling down the latus rectum of the parabola.

125. If chords be drawn from the extremity of the axis of a parabola, which is vertical, the velocities acquired by bodies falling down them are as the ordinates; and the times are in the subduplicate ratio of the abscissæ increased by the latus rectum.

126. If from any point in a rectangular hyperbola, whose axis is vertical, two lines be drawn to the extremity of the axis major, the times of descent down them will be equal.

127. If from the extremity of the major axis of an ellipse, which is perpendicular to the horizon, chords be drawn, making with the axis angles of 75° and 45°; and from the points where the chords meet the curve, ordinates be drawn to the axis; the square of the time down the first will be to twice the square of the time down the second, in the sub-duplicate ratio of the rectangles under the segments of the axis made by the ordinates.

128. If the resistance vary as the velocity, and

the force of gravity be constant, the times of describing all chords of a circle, terminating in the extremity of a vertical diameter, are equal.

---

129. From the middle point of a vertical line, standing on a horizontal plane, draw to the plane a straight line, such that if a heavy body begin to descend down it with the velocity acquired down half the given line, it may reach the bottom in the same time as it fell freely down the half of the given line.

130. Determine the position of a line drawn from a given point to a given inclined plane, through which a body shall be as long in falling as down the plane.

131. Determine the position of a line drawn from a given point to a given inclined plane, through which a body shall descend in the same time, as from the top of the plane to that point where the line meets the plane.

132. Determine the perpendicular height above a given inclined plane, through which a heavy body will be as long in falling, as it is in descending down the inclined plane if projected with the velocity acquired.

133. Determine a point in the altitude (produced) of a given inclined plane, from which if a line be drawn to the lowest point of the plane, a heavy body would descend down it in the same time as down the plane.

134. Determine a point in the length (produced)

of a given inclined plane, from which a heavy body would descend to the bottom of the plane, in the same time as another would fall freely through the altitude.

135. Determine a point in a given inclined plane, from which if a line be drawn to the extremity of the base, the time of a heavy body's descending down it may be equal to the time down the plane.

136. Determine a point in a given inclined plane, from which if a line be drawn to the extremity of the base, the time down it may be the least possible.

137. Determine a point in a given horizontal or inclined plane, to which if lines be drawn from two given points, the times of descent down them may be equal.

138. From a given point, without a given circle, draw a plane to the circle, so that the time down it shall be equal to the time down a given plane.

139. At the angular points of a given Isosceles right angled triangle, are three perpendiculars, of such length that the times in which a heavy body would descend down them freely, are in the proportion of the numbers 2, 3, 4; the shortest being at the right angle. Determine the length and inclination of a plane to be drawn from the extremity of each to a point within the triangle, so that a heavy body may descend down each plane in the same time.

140. Two points, A and B, being given in the same horizontal line; determine the position of

two planes, AC, BC, such that the time from A to B may be the least possible, supposing no velocity lost in passing from one plane to the other.

141. Determine the locus of all the points to which a heavy body will descend, in the same time, on straight lines drawn from two given points.

142. If a regular hexagonal canal be placed with two opposite angles in a vertical line, shew that the velocity acquired in falling from the highest to the lowest point, is to the velocity acquired in falling freely down the same height as $5 : 8$.

143. A body not affected by gravity, falls down the axis of a thin cylindrical tube infinite in length, the particles of which attract with a force which varies inversely as the square of the distance. Determine the velocity acquired in falling through a given space.

---

144. On an inclined plane of given length and height, it is required to assign a part equal to the height, through which a heavy body beginning its motion from the top of the plane, descends in the same time as it would fall freely down the height.

145. Determine a part of an inclined plane given in length and position, such that it may be equal to one given line, and the time down it may be equal to the time down another given line.

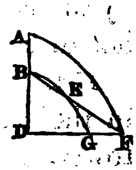
146. With a given straight line perpendicular to the horizon as radius, and the lower end as

centre, describe a circle, and draw a tangent at its extremity: then if any radius of this circle represent the position of an inclined plane, and a body be projected on it with the velocity acquired in falling through the given line; the space described on this plane in the time of its descent will be equal to the radius of the circle, together with the perpendicular drawn to the tangent from the intersection of the plane and the circle.

147. Two planes have a common base, and are inclined to the horizon at an angle of 30°. A non-elastic body is projected up one of them with a given velocity; it then descends and oscillates between them. Determine the whole space described and the time of motion.

148. Two equilateral triangles are placed with their bases at a given distance from each other upon the same horizontal line; and a non-elastic body falls down the side of the first, moves along the space between the bases, and up the side of the second triangle, the vertex of which it just reaches: having given the side of the first triangle, determine that of the second; and also the whole time of the motion.

149. If two equal parabolas be placed with their axes in the same vertical line ABD; and a heavy body fall down the plane BEF, and the ordinate FD cut BE in G; shew that the time down BE is to the time down EF :: DG : GF.



150. The plane of a cycloid whose axis is equal to (a), is inclined to the horizon at an angle of 60°. Determine the time of descent down a chord drawn from the vertex to the extremity of the base.

151. If a body describe from rest any part of a curve AB on an inclined plane, the velocity at B will be equal the velocity acquired in falling through a vertical space equal to the perpendicular depth of B below A; and the time of describing AB along the plane is to the time of describing AB (were the plane vertical) : : $\sqrt{\text{sin. of inclination}}$: radius.

152. A cone whose altitude is (a) feet revolves about its axis with a velocity of (n) feet *per* second at the circumference of the base; but in the time of one such revolution a body would descend down its slant side when the cone is at rest. Required the radius of the base.

153. If at the same instant that a right-angled triangle, whose base is perpendicular to the horizon, begins to revolve about the perpendicular with a given velocity, a heavy body begins to descend down the hypothenuse by the force of gravity; determine the length of the path described by the body in its descent; supposing the descent not to be affected by the rotation of the triangle.

154. Two equal balls of known weight descend in the same time, one freely from the top of a tower, and the other on an inclined plane; and they arrive also at their respective bottoms in the same time, when the sum of their momenta is

equal to (m^2). The rectangle under the height of the tower and the length of the plane being given; determine the length and height, the time of descent, and the momentum of each ball.

155. A body falls through a sphere in the chord *AB*. Determine the ratio of the time of describing *AB* to the time of revolving at the surface.

156. A body falls down the axis of a parabolic vessel; at what point of its axis will it have acquired a velocity sufficient to make it revolve in the corresponding circle without either ascending or descending?

---

157. Given the base, to determine the altitude of an inclined plane, when a body falling down it strikes another horizontally with the greatest velocity possible.

158. Given the length of an inclined plane to determine its elevation, when the horizontal velocity is the greatest possible.

159. Of all inclined planes having the same base, determine that which will be described in the least time.

160. Given two sides of a triangle; determine the third, so that the time down it may be a minimum.

161. Given the sum of the two sides of a right-angled triangle; construct it, so that when it is placed in a vertical plane with one side parallel to the horizon, a heavy body may descend down the hypothenuse in the least time possible.

162. Given the base and the sum of the sides of a triangle; determine the triangle when the times of the body's descending down the sides, taken together, may be a minimum.

163. Determine the line of quickest descent from a given point to a straight line given in position; or from the given line to the given point.

164. Determine the line of quickest descent from a given point to the circumference of a circle given in magnitude and position, the point being either within or without the circle: and also from the given circle to the given point.

165. Determine the line of longest descent from a given point to the circumference of a circle given in magnitude and position: and also from the given circumference to the given point.

166. Determine the lines of quickest and slowest descent betwixt the circumferences of two circles given in magnitude and position, one of them above and the other below a horizontal line.

---

167. A parabola is placed with its axis vertical. Draw the line of quickest descent from the curve to the focus.

168. A given parabola is placed in a vertical plane with its axis parallel to the horizon. Determine a point in the curve, from whence a heavy body will descend from rest along a straight line to a given point in the axis in the least time possible.

169. If a parabolic segment made by a line

passing through the focus be situated with the line vertical; shew that the line of quickest descent to the vertex of the parabola from the highest point of the segment will be a tangent at that point.

170. Determine the straight line of quickest descent from a given point in the axis major of a given ellipse, supposed to be perpendicular to the horizon, to the curve.

171. Determine the straight line of quickest descent from a given point in the transverse diameter of a given semi-ellipse; its semi-conjugate making a given angle with the horizon.

172. A cycloid being placed with its base horizontal and vertex downwards: determine the straight line of shortest descent from any point in the base to the curve.

173. A cycloid is placed with its base AB horizontal and vertex C downwards. To what point of the curve must a tangent FDE be drawn meeting AB produced in F, and a tangent at the vertex CE, so that the time down FDE, together with the time of moving along EC with the velocity acquired may be a minimum?

174. If a quadrant of a circle be placed in a vertical plane with its upper radius parallel to the horizon; determine in what direction a line must be drawn from a given point in that radius, so that a heavy body descending along it may arrive at the arc in the least time possible.

175. Having given the segment of a circle in a vertical plane, whose chord is parallel to the ho-

rizon: determine a point in the arc, such that a heavy body may descend from it down a right line to a given point in the chord in the least time possible.

176. If a quadrant of a circle be placed in a vertical plane with its upper radius *OB* parallel to the horizon; determine a point *C* in the quadrantal arc, to which if a tangent be drawn meeting *OB* produced in *D* and a tangent at the vertex *AE*; the time down *DE* together with the time of moving along *EA* with the velocity acquired may be a minimum.

177. If a quadrant of a circle be placed in a vertical plane with its upper radius parallel to the horizon; shew that a heavy body will be longer in descending down the chord of the whole arc than in falling down two chords drawn from its extremities to any given point of the arc; no velocity being lost in passing from one chord to the other.

178. A given circle is placed in a vertical plane; determine a point in a straight line given in position in the same plane from which if a heavy body be let fall down a vertical line cutting the circle, the time of passing through the chord may be a maximum.

179. If the plane and axis of a cissoid be vertical; determine the line of quickest descent from the curve to the farther extremity of the diameter.

180. Two points being given in the same vertical plane, but not in one horizontal line; it is required to determine the position of two in-

clined planes such, that the time of descent from the higher point to the lower on these two planes shall be less than on any other two inclined planes whatsoever.

181. The same being given; determine the position of three inclined planes such, that if no velocity be lost in passing from one to the other, the time of descent may be less than the time on any other three planes, when the line of descent is of a given length.

182. The upper extremity of an inclined plane being given; determine its position, so that the time shall be a minimum in which a body falls down it, and afterwards moves to a given point in the horizontal plane with that part of its acquired velocity which is not destroyed by its impact on that plane.

183. Determine the motion of a given heavy particle projected along a rod which is supported on a fulcrum at its middle point, and is of a given uniform density; the motion commencing from the middle point, and the first position of the rod horizontal.

184. If a given heavy ball be laid upon one edge of a horizontal plane of an indefinite length, and round this edge the plane be made to revolve downwards with such an uniform motion that the angle of inclination may increase at any given rate. Determine what length the ball will descend along the plane before it acquires such a velocity as will cause it to fly off and cease touching the plane.

185. If two attractive spherical bodies of the same density and of given unequal diameters be placed at a given distance from each other in a medium of the same density as our atmosphere, and then move towards each other, unaffected by any force but that of their attraction: determine how far each will move before they impinge upon each other, as also the time they will be in motion; supposing the force acting on the least body at the commencement of the motion to be such, as if uniformly continued would move it over a space of $16\frac{1}{12}$ feet in one second.

186. If a plane touch the surface of the earth (supposed to be a sphere) at a given point P, from which a body is projected in the direction of the plane, and slides thereon with the initial velocity of (v) feet *per* second; determine what distance it will have gone from P when its motion is destroyed, supposing it to be acted on by no force but the earth's gravity.

SECTION IX.

1. If a body be projected at an angle of 15°, and the horizontal range be 841 feet. Determine how high the body would rise, if it were projected straight upwards with the velocity of projection.

2. Given the time of flight of a projectile; determine its greatest height above the horizontal plane passing through the point of projection.

3. If the horizontal range of a projectile be to the greatest height to which it rises above the horizontal plane :: $4 : \sqrt{3}$; determine the angle of projection. Determine the angle, also, when the ratio is that of $2 : \sqrt{3}$.

4. If the velocity of projection be given, and the area of the parabola described be one third of the area of the square described on the horizontal range; determine the angle of projection.

5. If the velocity of projection be given, and the area of the parabola described be a maximum; determine the angle of projection.

6. Given the velocity and direction of projection; find the direction and velocity of a projectile at the end of (t''); and its greatest height above the horizontal plane passing through the point of projection.

7. Two bodies are projected from two given points, in given directions, and with given ve-

locities. Determine their distance at the end of (t'').

8. Two bodies are projected from the same point, with the same velocities, towards the same mark situated in a horizontal plane. The angles of elevation are as 2 : 1. Compare the areas of the parabolas described.

9. A body projected from one extremity of the diameter of a circle, at an angle of 45°, strikes a mark placed in the centre. Determine the velocity of projection, and the greatest altitude to which the body rises.

10. Determine the velocity and time of flight of a body projected from one extremity of the base of an equilateral triangle, and in the direction of the side adjacent to that extremity, towards an object placed in the other extremity of the base.

11. Determine the time of flight of a body projected at an angle of 30°, with a velocity of (a) feet in a second; the mark being in a plane inclined at an angle of 60°.

12. Determine the same, when the angle of projection is 60°, and the angle of the plane's inclination is 30°: and what will be the greatest elevation of the body above that plane?

13. A body projected at an angle of ($a°$), hits a mark at the distance of (b) feet, on an inclined plane, whose elevation is ($c°$). Determine the velocity of projection, the greatest height above the plane, and the time of flight.

14. In what direction must a body be projected with a given velocity from a point in a given in-

clined plane, that the range may be the greatest possible?

15. Given the height and breadth of a house; investigate the equation, from which may be determined such an elevation of its roof, that a body beginning to descend from its summit, may fall upon the ground at the greatest possible distance from the bottom of the house.

16. A body is projected from the top of a given inclined plane: determine the direction of projection, in which the least velocity will bring it to the bottom of the plane. Determine also the velocity.

17. A body is projected, at a given angle, from the top of a tower, with a given velocity. Determine (geometrically) the range.

18. Determine (geometrically) in what direction a body must be projected from the top of a tower, with a given velocity, so that it may fall at the greatest distance from its bottom.

19. A ball projected from the top of a tower, at an elevation of 30° above the horizon, fell in (t''), at a distance of (a) feet from the base. Determine the height of the tower.—Also, if the angle of elevation be 45°, and (a) the height of the tower; determine the distance the ball falls from the base.

20. If a body be projected from the top of a tower horizontally, with a velocity acquired in falling down a space equal to the height of the tower; at what distance from the base will it strike the horizontal plane?

21. If two bodies projected from the top of a tower with different angles of elevation, and different initial velocities, strike the horizontal plane at the same point. Determine the height of the tower.

22. With what velocity must a body be projected from a tower, in a direction parallel to the horizon, so that it shall strike the ground at a distance from the foot of the tower equal to half its height?

23. A person at a given distance (a) from the bottom of a vertical line, projects a ball at an angle of 45°, which just touches the top, and then falls at the distance (b) from the bottom, on the other side. Determine the height of the line.

24. Given the greatest range of a projectile on a horizontal plane; determine at what distance from the point of projection an object, whose perpendicular height is (d), must be situated, so that the projectile may just strike the top of it.

25. Given the horizontal range of a body projected with a given velocity, at an angle of 30°: determine the elevation, when a body is projected, with the same velocity, from a point (a) yards above the level of the horizon, so that it may fall at the greatest distance possible; which distance also is required.

26. Given the velocity of sound; determine the horizontal range, when a ball, at a given angle, is so projected towards a person, that the ball and the sound of the discharge reach him at the same instant.

27. A shell being discharged at a given angle, the sound of its explosion was heard at the mortar (t'') after the discharge. Having given the velocity of sound, determine the horizontal range of the shell.

28. A shell was discharged from a mortar, which in its flight just touched the top of a vertical line, and in (m'') after fell at the distance of (b) feet from the bottom of the line. The report of its explosion was heard at the mortar (n'') after the explosion. Determine the height of the vertical line.

29. A shell was discharged from a mortar to hit an object at a given distance. At what distance from the object, in a line perpendicular to that joining the mortar and object, must a person be placed, so as to hear the report of the discharge and the explosion of the shell at the same instant of time, allowing the velocity of the ball to be to that of sound as $m : n$.

30. A ball is projected from a given point in the horizontal plane at an angle of 30°, and after describing two-thirds of its horizontal range, strikes against a sonorous body. Having given the time elapsed between the instant of projection, and that when the sound reaches the point of projection, determine the initial velocity.

31. A body is projected from the summit of a mountain of 30° elevation, so as just to strike at the bottom, and with double the velocity of projection, which is what would have been acquired in falling vertically down (a) yards. Determine

the height of the mountain, and the greatest altitude to which the projectile rises.

32. A ball being shot at a given angle from a cannon, whose greatest horizontal range is (a), struck a vertical object standing at such a distance, that if it had been projected against it in a direction perpendicular to its surface, its force would have been (m) times as great as it then was. Determine the distance of the object from the cannon.

33. Given the velocity of projection: determine (geometrically) the direction, so that the sum of the horizontal range and the greatest height to which the body rises above the horizontal plane may be the greatest possible.

34. Shew that the velocity being given, the range upon any plane is the greatest when it passes through the focus of the parabola described. And the locus of all the greatest ranges will be a parabola.

35. Given the point of projection, determine the velocity and direction of projection, so that the projectile may pass through a given point, and strike the horizontal plane in another given point.

36. Given the velocity and direction of projection; determine (geometrically) where a body will strike a given inclined plane, not passing through the point of projection.

37. If a body be projected with a given velocity; determine the angle of elevation, that the length of the curve described in its flight above

a horizontal plane may be the greatest possible.

38. Two bodies are projected towards each other, at the same given angle, in the same vertical plane, from two given points, so as to describe the same parabola. Determine the point where they will meet; and the velocities of projection and impact.

39. If in the last problem, the bodies be perfectly elastic, and $A = 3B$, determine the respective paths of the bodies, and of the common centre of gravity *after* impact; and the area *contained* between the paths described by B *before* and *after* impact.

40. Two balls are projected at the same instant from two given points in a horizontal plane, and in opposite directions, so as to describe the same parabola. What must be their relative magnitude, and their elasticity, so that after impact one of them may return through the same path as before, and the other descend in a right line?

41. If two bodies be projected from the same point with equal velocities, and in the same vertical plane; determine the angles of elevation, so that the greatest area may be contained between the parabolas described by the bodies; the horizontal range being the same.

42. Two bodies describe equal parabolas, one upon an inclined plane, and the other acted on by the whole force of gravity. Compare the velocities of projection.

43. If any number of bodies be projected from

a given point, with a given velocity, in different directions: determine the locus of the greatest heights to which they will rise.

44. If any number of bodies be projected at the same instant, from the same point, and with equal velocities, but in different directions in the same vertical plane, they will, at the end of any time, all be found in the circumference of some circle.

45. If any number of bodies be projected from the same elevated point, and with the same velocities, in a horizontal direction; the locus of the points in which they will strike a given inclined plane will be a conical frustum.

46. Determine the locus of the points from which a body projected against a given point shall return to the hand.

47. Determine the nature of the curve which shall cut all the parabolas that can be described by a body projected from a given point with a given velocity, so that the time of flight from the given point to the curve may be the same in each parabola.

48. The inclination of a perfectly smooth bank to the horizon is 30°, and a body is projected up the bank in a direction making an angle of 45° with the intersection of the bank and the horizontal plane. Determine the curve described by the body, and the point where it will again meet the horizon.

49. If the elasticity of a body be to perfect elasticity :: e : 1; determine the point in a given

horizontal line from which it must fall, so that after striking a given inclined plane, it may hit a given object.

50. If a body falls from a given point A, and impinges upon an inclined plane; determine where, after reflection, it will again meet the plane.

51. If a perfectly elastic ball fall from a given height above the plane of the horizon, and impinge upon a plane inclined at a given angle to the horizon; determine on what point it must strike, so that it may fall on the horizontal plane at the greatest distance possible from the bottom of the inclined plane.

52. From what height above a given point must an imperfectly elastic ball be let fall, so that impinging on a horizontal plane and arriving at the given point, the whole time of motion shall be the least possible.

53. A perfectly elastic ball having fallen freely through a given space impinges on a non-elastic immovable plane. Determine the inclination of this plane to the direction of its motion, so that the ball after reflection by it may hit a given mark.

54. A perfectly elastic body descends from a point in a vertical line in (n) seconds, and after impinging on a non-elastic inclined plane, whose perpendicular height at the point of impact is (a) feet, falls at the foot of the plane. Determine the inclination and length of the plane, supposing the point of impact to be (b) feet from the top.

55. Determine the chord of a circle drawn from the upper extremity of the vertical diameter, down which an elastic ball descending, and being reflected by a horizontal plane passing through its extremity, shall describe the greatest range on that plane.

56. With what velocity must a perfectly elastic ball be projected up a given inclined plane, that being reflected by the perpendicular it may come into the hand again?

57. Determine the motion of a ball projected up an inclined plane with such a velocity as to fly over the top of it. Shew that the parabola which it describes, has the same directrix as that in which it would have moved, had the inclined plane not existed.

58. Given the base of an inclined plane; determine the plane such that a body projected directly up it with a given velocity, may, after passing the top, hit a given mark.

59. Determine the length of an inclined plane, whose height is half its base, so that a body projected directly up the plane with a given velocity may be as long after leaving the plane before it again meets the horizon, as it was in ascending the plane. Determine also the range and the time of flight.

60. Given the base of an inclined plane; determine its inclination, so that a body projected directly up it with a given velocity, may, after passing the top, fall at the greatest possible distance beyond the base in the same horizontal line.

61. If a body be projected up a plane *AC*, inclined at an angle of 45° to the horizon, with the velocity acquired in falling down a vertical space equal to *AC*: determine the range on the horizontal plane passing through the point *A*. Determine also the time between its leaving the point of projection and meeting the horizontal plane.

62. A ball whose elasticity is to perfect elasticity as $e : 1$, is projected with a given velocity in a direction making an angle of 60° with the horizon; and when at its greatest height is reflected by a vertical plane. Determine where the ball will again strike the horizon, and the whole time of flight.

63. A body half elastic moving along a horizontal plane is reflected by a hard plane inclined at an obtuse angle to its course; prove that the time of flight on the inclined plane is to the time of acquiring the velocity before impact by a body descending vertically, as the tangent of the plane's inclination is to the radius.

64. A body is projected perpendicularly upwards with a velocity that would carry it to a given point *C*; but at an intermediate point *B* it meets a plane inclined to the horizon at an angle of 45°; determine the point on the horizontal plane where the body, which is perfectly elastic, will fall.

65. Determine the least velocity with which a body projected from the top of a tower of given height, after reflection from the horizontal plane,

shall strike the top of another tower, whose distance and height are given. Determine also the direction, and the time of flight.

66. A ball, whose elasticity is to perfect elasticity as $e : 1$, is projected obliquely upwards from a point in the horizontal plane, upon which it impinges and rebounds continually: Prove that the ranges and times of flight in the successive parabolas described, form geometric progressions, and find their sum.

67. An elastic ball being projected obliquely upwards is continually reflected by a non-elastic horizontal plane; and the sum of the areas of all the parabolas described is to the area of the first as $8 : 7$. Determine the elasticity of the ball.

68. If bodies, which are perfectly elastic, be let fall from different points in the same horizontal line, and impinge upon an inclined plane given in position; determine the locus of the foci of all the parabolas that can be described by the bodies after reflection.

69. A perfectly elastic ball projected from the top of a given vertical plane AB strikes a parallel plane CD, and being reflected descends to the horizontal plane BD, from whence it is again reflected to A. Having given the velocity and direction of projection, determine the distance BD between the vertical planes.

70. A circle has chords drawn from the extremity of its diameter which is at right angles to a horizontal plane. Determine that chord down which an imperfectly elastic body descending and

reflected by the horizontal plane may describe the greatest horizontal range; and knowing the degree of elasticity, find the range.

71. A perfectly elastic body descending down the chord AC drawn from the upper extremity of the vertical diameter AB of a circle, is reflected by the plane CB, and describes its path as a projectile. Shew that this path strikes the circle at the opposite extremity of the diameter CD.

72. Prove that the elevation at which a projectile must be thrown from a plane of given inclination (I) in order that the range may be (n) times the space due to the velocity of projection, is determined by the two values of E in the equation, $\sin.(I+2E)=\sin. I+\frac{n}{2}.\cos.^2 I$. Compare also the times of flight and the greatest heights for the two values of E thus found, where the plane is horizontal and $n=1$; and where $I=30^\circ$, and $n=\frac{4}{3}(\sqrt{3}-1)$.

73. From what height must a perfectly elastic ball be dropped on the convex surface of a given hemisphere, so that after reflection it may describe the greatest possible horizontal range?

74. A body is projected, with a given velocity and at a given elevation above the horizontal plane, from the summit of a hill which is of the form of an upright paraboloid. Supposing the parameter known, determine where the projectile will strike it.

75. Given the base and altitude of a paraboloid,

from the top of which a body is projected with a given velocity. Determine the angle at which the body must be projected, so that it may fall at the greatest distance possible from the place of projection.

76. A hollow paraboloid is placed with its vertex downwards, and axis vertical. Determine (by geometrical construction) a point in any given diameter of the terminating circular plane, such that a body let fall from it on the surface of the paraboloid may after one rebound hit the vertex.

77. If perfectly elastic balls be let fall from the directrix of a parabola whose axis is vertical, and be reflected from the curve, determine the locus of the vertices of the parabolas described.

78. In the last problem determine the locus of the extreme ranges upon the tangents at the points of incidence of the reflecting curve.

79. If a body be projected at an angle of 45° with the velocity acquired down the axis of a cycloid: compare the time of flight with the time of an oscillation.

80. If DBV be a cycloid whose base is horizontal and vertex downwards: determine from what height WB a body must fall on the given point B, that it may be reflected into the vertex.

81. AVB is a cycloid whose base AB is horizontal, and vertex V downwards. If EOF be drawn parallel to the base and bisecting the axis in O; and at E and F two vertical hard planes be placed; shew that a body descending down the

curve from E with the velocity acquired in falling down a vertical space EO, will by being reflected at F and E continually, describe the same path: and find the interval between its leaving E and returning to it, the body being perfectly elastic.

82. If a heavy body descend freely by the force of gravity upon a common cycloidal curve, the radius of whose generating circle is given, and its distance from the vertex of the cycloid be (n) feet at the commencement of the motion; determine where it will quit the curve, its distance from the axis when it impinges upon the horizon, and the time of its whole descent.

83. The middle of a tube of given length is fixed to the top of a vertical line of given altitude above the horizontal plane. Determine its position so that a ball put in at its upper end and descending by the force of gravity may strike a point on the horizontal plane at the distance of (a) feet from the bottom of the line; and find what is the greatest distance it can strike.

84. If a body be projected from the earth's surface, in a direction making an angle of 45° with the horizon, and with the velocity of a body revolving in a circle at the surface: determine the point at which it will reach the earth again, and the time of motion.

85. If a body be projected at an angle of 45° with the horizon, and an arc of the earth's surface intercepted between the points of projec-

tion and incidence be 60°; determine the velocity of projection.

86. Under what law of gravity will a projectile describe a curve expressed by the equation $a^3x^2 = y^5$, in a non-resisting medium?

SECTION X.

1. What must be the length of a pendulum which vibrates seconds at the distance of (n) radii from the surface of the earth?

2. In what time would a pendulum, 80 inches long, vibrate at the distance of two radii above the earth's surface?

3. How far within the earth's surface must a pendulum, whose length is (l), be carried, to vibrate (m) times in (n) seconds?

4. What must be the length of a pendulum vibrating seconds at the distance of half the radius from the earth's centre?

5. If a pendulum vibrate seconds at the earth, it would vibrate minutes at the moon, the distance of the moon being taken equal to 60 radii of the earth.

6. The length of a pendulum oscillating seconds on the earth's surface being given; determine the length of a pendulum oscillating seconds at the distance of the earth's radius from the surface; and determine a point below the earth's surface, where the last pendulum will vibrate in the same time.

7. A pendulum, whose length is (l), at the distance of (n) radii from the earth's centre, vibrates in (t) seconds: to what point within the earth

must a pendulum, whose length is (l), be carried, to vibrate in (t) seconds?

8. How long must a pendulum be to vibrate three times, whilst a heavy body falls through 81 feet?

9. A heavy body is dropped into a well, and before it reaches the bottom, a pendulum, whose length is (l) inches, vibrates (n) times. Determine the depth of the well.

10. A pendulum, whose length is (l), makes (n) vibrations between the time of seeing the flash and hearing the report of a cannon. Determine the distance of the cannon from the observer.

11. If a clock loses or gains (n) seconds in a day; investigate an expression for the alteration which must be made in the length of the pendulum.

12. A clock which kept true time on the earth's surface, being carried to the top of a mountain, lost (n) seconds in a day. Determine the height of the mountain.

13. The pendulum of a clock, which gained (n) minutes *per* day, being lengthened (a) inches, lost (m) seconds an hour; determine what ought to be the true length; and how often it will vibrate in a minute.

14. The sum of the vibrations made by three pendulums in (n) seconds is (s), and the ratio of the numbers of vibrations made by each, as a, $a+b$, $a+2b$; determine the lengths of these pendulums; the length of the seconds' pendulum being (l.).

15. Find the radius of a circle in the chord of which a pendulum would vibrate twice in a second.

16. The times of vibrations of two pendulums of the same length at two different places are as $m : n$. Determine the proportion of the force of gravity at those places.

17. The number of vibrations of two pendulums, whose lengths are (L) and (l), in the same time, at different places, are as $m : m+n$; determine the proportion of the force of gravity at those places.

18. If a clock, at a place A, on the earth's surface, keeps true time, but when taken to another place B, loses (n) minutes every day, but goes right if shortened $(\frac{1}{m})^{th}$ of an inch: compare the force of gravity at A and B.

19. Find the length of a pendulum which vibrates 36 times in one minute; and the distance from the surface of the earth at which it will vibrate 24 times in a minute; the force of gravity varying inversely as the square of the distance from the earth's centre.

20. Find the length of a pendulum which will vibrate exactly as many times in a minute as it is inches long; the length of one vibrating seconds being supposed unknown.

---

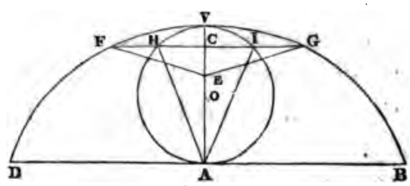
21. Divide the arc of a cycloid into four equal parts.

22. Divide the arc of a cycloid into two parts, so that the time of a body's oscillating through them may be in the ratio of 1 : 5.

23. If the arc of a common cycloid, in which a pendulum oscillates, be divided into four equal arcs, the time of moving through the first quarter is equal to one-third of the whole time of oscillation.

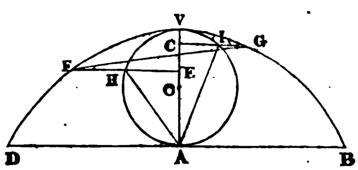
24. If the generating circle of a cycloid move with half the velocity due to its diameter, the describing point will move in the same manner as a body oscillating in the cycloid.

25. If in the axis of a common cycloid *DVB*, there be taken two points *C* and *E*, equidistant from the vertex and the centre of the generating circle; and *FCG* be drawn parallel to *DB*,



and *EF*, *EG* joined. Shew that the sector *EFVG* is quadrable, and equal to the triangle *AHI*.

26. If from any point *F*, in the curve of the cycloid, a perpendicular to the axis *FE* be let fall, and *VC* be taken equal to *OE*, and *CG* drawn perpendicular to the axis, and *FG*, *HA*, *AI*, be



joined; shew that the area *FGV* is equal to the sum of the triangles *AHE*, *AIC*.

27. In the case of the preceding problem, shew that if FE cut the axis in the middle point between O and V, the area FGV is equal to half the area of the hexagon inscribed in the circle. But if E fall in the centre, the area is one-fourth of the inscribed square.

28. If the bases of two equal cycloids be parallel, and the vertex of one in the base of the other; prove that the angle formed by the intersection of the curves will be a right angle.

29. Required the locus of the intersections of tangents to the cycloid and corresponding points of the generating circle, determined by drawing ordinates to the axis.

30. When a pendulous body oscillates in a cycloid, beginning at the highest point; compare the tension of the string at the lowest point, with the tension caused by the natural weight of the body.

31. Determine in what point of the cycloid, the force stretching the string, and the force accelerating the body, will be equal.

32. Determine (by geometrical construction) that point in a cycloid at which the velocity of an oscillating body is half the greatest velocity.

33. A body descends down the arc of a cycloid, whose base is parallel to the horizon, and vertex downwards; determine at what point its vertical velocity is the greatest.

34. If ABV be *part* of a cycloid, whose axis is DV; and a body begin to descend from A, and from A and B the horizontal lines AE, BF, be

drawn, meeting the axis in E and F, and on EV a circle be described, cutting BF in x; shew that the time down $AB \propto Ex$.

35. If a body begin to move from the highest point of a cycloid, whose base is horizontal and vertex downwards; shew that the time of describing any arc $AP \propto$ that part of the base which is intercepted between A and a perpendicular to the curve at P.

36. The time of a body's falling down a semi-cycloid, is to the time down its chord as half the base of that cycloid is to the chord. Required a demonstration.

37. Compare the time of a body's descending down *any* arc of a cycloid with the time down the corresponding chord, the motion beginning from the highest point.

38. What chord in the cycloid will be described by a heavy body, whilst another heavy body descends down the semi-cycloidal arc, the motion of each commencing at the extremity of the base, which is parallel to the horizon?

39. If a body moves in an inverted cycloid; determine its position when half the time in which it would move from the highest to the lowest point has elapsed.

40. Determine the length of a pendulum, which beginning an entire oscillation in a common cycloid, from a given point, shall arrive at a given perpendicular to the horizon in less time than any other beginning an oscillation from the same point.

41. An inclined plane is a tangent to a cycloid, at the middle point between the highest and the lowest points: determine what must be the length of the plane, that the time of falling down it may be equal to the time of half an oscillation.

42. A body, which is half elastic, descends along the arc of an inverted cycloid, and is reflected by the axis, which is vertical. Determine the space described in the time of (n) oscillations of a pendulum vibrating in the same cycloid.

43. A body moves in a cycloid, the plane of which is inclined to the horizon at a given angle; determine the time of an oscillation; and the point where the velocity perpendicular to the horizon is a maximum.

44. In a circle, compare the time of falling down a chord of 60°, to the extremity of a diameter perpendicular to the horizon, with the time of the oscillation of a pendulum equal in length to the chord.

45. Determine in what arc of a circle a pendulum must vibrate, so that the time of the whole descent may be equal to the time in which a heavy body would fall along the chord of the same arc.

46. Compare the time of an oscillation in the chords of a circle with the time of an oscillation in a cycloid.

47. If a given pendulum be made to oscillate in a cycloid and in a circle, its greatest velocity in the cycloid is to its greatest velocity in the circle, as the cycloidal arc described in its descent is to the chord of the circular arc described.

48. If two bodies, whose weights are as 2 : 3, oscillate, one in a semicircle, and the other in a cycloid, the whole tensions of the two strings at any given inclination of them to the horizon, will be equal; the motion in each case beginning from the highest point.

49. A body suspended by a string oscillates on an inclined plane through a semicircle whose diameter is horizontal. Determine the tension of the string at any point; and the law of its variation at any given point for different inclinations of the plane.

---

50. A chandelier, suspended from the top of a church, vibrates (n) times an hour; and the height of the church is divided by the chandelier in extreme and mean ratio. Determine the height, and the distance of the chandelier from the ground.

51. If at the same instant that a stone is let fall from the top of one of two towers, whose distance is (a) feet, a pendulum begins to oscillate at the other, the length of the string being equal to the height of the tower: and the observer at the pendulum hears the beat and the fall of the stone at the same time; but the observer at the top of the first tower hears the stroke of the stone twice as soon as the beat of the pendulum. Determine the height of the towers.

52. Two pendulums whose lengths are L and l, begin to oscillate at the same time, and are again

coincident after (n) oscillations of the former pendulum. Having given L, it is required to determine l.

53. Given the length of the thread by which a ball of given weight is suspended, and the arc it describes in vibrating: determine the greatest horizontal force with which it acts on the centre of suspension.

54. If a body suspended by a string oscillates through a quadrant, the extremity of the quadrant being the lowest point; prove that the force stretching the string at the lowest point is three times that which is due to the weight of the body.

55. If a given cone suspended by its vertex be made to vibrate with its axis in a vertical plane; determine the force exerted on the centre of suspension at any given position of the cone.

56. Find the ratio of the times of oscillation of a pendulum at the pole and at the equator, supposing the earth a sphere.

57. From the poles to the equator, the decrease of the length of a pendulum always vibrating in the same time $\propto \cos.^2$ of latitude. Required proof.

58. In a given latitude a pendulum will oscillate once in a second, supposing the earth not to revolve round its axis. Determine the angular motion round the axis that the pendulum may oscillate once in two seconds.

59. If a pendulum whose length is (l) inches, would oscillate in one second at the pole of a sphere whose radius is (r), what must be the time

of rotation round the axis, so that the same pendulum at the equator may oscillate twice in three seconds?

60. If the earth's motion about its axis were to cease, how much would a clock keeping true time in a given latitude gain in 24 hours?

61. Determine the pressure upon the axis of any vibrating body in any given position.

62. Determine the correction due to the length of a pendulum for the thickness of its axis.

63. If a pendulum vibrating through an arc of 2° on each side of the vertical, keep true time, find nearly the error of time introduced by making it vibrate through $2^\circ\ 10'$.

---

64. If any number of bodies be retained in horizontal circular orbits by means of strings of unequal lengths, and the distances of the centres from the point of suspension be equal; the times of their revolution will also be equal.

65. If a pendulum be made to revolve in a conical surface; find the absolute time of a revolution.

66. A weight suspended by a string of given length is whirled round, so as to describe parallel to the horizon the base of a cone, of which the vertex is the point of suspension. The tension of the string being three times the natural weight of the body, in what time does it revolve?

67. If in the inner surface of a paraboloid, a

body whirled round as a circular pendulum (the point of suspension being in the axis) describe a circle; then the time of revolution is the same, whatever be the radius of the circle; and it is equal to two oscillations of a common pendulum, the length of which is half the parameter of the parabola.

68. If a body fall down the axis of a right-angled cone, whose vertex is downwards and axis vertical; in what point of its descent will it have acquired velocity sufficient to describe a circle on the surface of the cone, when whirled round as a circular pendulum?

69. If a body, whirled round by a string in a vertical plane, just keep the string extended at the highest point; determine the proportion of the centrifugal force to the weight at the lowest point.

70. If a body whirled round by a string describes a circle in a vertical plane; having given the length of the string and the periodic time, compare the force of tension with that of gravity, and shew that the string cannot retain the body in the circle, unless it can support six times the weight of the body.

71. A string of given length will just support a given weight (w). If a less weight (w') be fixed to the end of it and whirled round in a vertical plane; determine the time of a revolution when the tension of the string is a maximum.

72. Let two weights fastened to two equal strings (l, l) move in a vertical plane, the one os-

cillating, the other revolving, and at the lowest points let the velocities due to the heights, h, k', be such that $\frac{h}{2l}=\frac{2l}{h'}=m^2$; then prove that the time of the revolution of one weight is to the time of oscillation of the other as $m:1$.

---

73. At what distance from one of its extremities must a slender rod of uniform thickness and given weight be suspended, that it may oscillate in a given time?

74. Determine the length of a pendulum that shall vibrate in the same time as a cylindric rod that is $(a+b)$ inches long, and suspended at the distance of (b) inches from one of its ends. Determine at what point in the axis the point of suspension must be, when the time of vibration is the shortest.

75. A thin rod of given length and uniform thickness oscillates seconds when suspended from its extremity: at what point must it be suspended to oscillate once in (n) seconds?

76. A straight rod is suspended at a point distant $(\frac{1}{4})^{\text{th}}$ of its length from its extremity. Compare the time of its oscillation in this case with the time of its oscillation when suspended at its extremity.

77. Find the time of vibration of a cylindrical rod of given length suspended at one end, the density of which varies as the distance from the point of suspension.

78. A cylindrical rod of given weight and length, when suspended at one extremity, vibrates seconds. At what point of the rod must a given weight (w) be suspended that it may oscillate (n) times in a second?

79. SA is a rod of given length without weight, at the extremity of which a given weight (w) is suspended: determine at what point a given weight (w') must be suspended so that the whole may vibrate in the least time possible.

80. A pendulum of given length, is suspended (n) inches below its upper end, with a weight (w) connected to that end. Determine what weight must be fixed to the lower end, so that in vibrating its momentum may be the greatest possible.

81. Two particles of matter are attached at different points to an inflexible line without weight, which is suspended by its extremity. Determine the time of a small oscillation, supposing one of them to be deprived of all its inertia, and the other of all its weight.

82. Determine the distance of the point of suspension from the centre of gravity of a given system of bodies, that the time of an oscillation may be the least possible.

83. A slender rod of uniform thickness revolves round an axis passing through one of its extremities: determine at what point an obstacle must be opposed to it, that there may be no stress on the axis from the shock. Determine also what quantity of matter should be collected in this

point that the impulse on the obstacle may be the same as that of the rod: also at what distance from the axis the obstacle must be opposed that the impulse may be the same as if the whole matter in the rod were collected in that point.

84. Two straight rods equal in length are suspended by their extremities, one being of uniform density, and the density of the other varying as the $(n)^{\text{th}}$ power of the distance from the point of suspension, and they make small oscillations in times which are as $\sqrt{5} : \sqrt{6}$. Determine the value of (n).

85. Two equal weights are fixed, one at the middle point and the other at the extremity of an inflexible rod, supposed to be without weight, and suspended from the other extremity. If this compound pendulum be made to vibrate through small arcs, determine the time of its vibration.

86. A thin rod of given length weighing one pound is suspended from its upper extremity, and has a weight of half-a-pound attached to it at one-third of its length from the point of suspension. Compare the time of its oscillation with this weight attached, with the time of its oscillation without it.

87. If a cylindrical rod whose length and weight are given, be suspended at a distance (d) from one of its ends, and a weight (w) be attached to the lower end; what weight must be attached to the upper that the time of an oscillation may be the same as before.

88. Determine the dimensions of a cone, whose

solidity is given, which if suspended by its vertex shall vibrate as many times in a minute as it is inches long.

89. Of all cones of a given convex surface, the base excluded; determine that which being suspended at its vertex shall vibrate in the least time.

90. Determine the ratio of the diameter of the base to the altitude of a cone, so that the centre of oscillation, when suspended by the vertex, may be in the centre of the base.

91. Determine the centre of gyration of a right cone revolving about an axis passing through its centre of gravity parallel to one of its sides.

92. Two globes, of equal weights and diameters, are placed in contact on a straight rod, which passes through both their centres; the point of suspension is distant one diameter from the vertex of the upper one. Determine their common centre of oscillation.

93. A sphere of given radius is suspended by a point at a distance from its centre equal to its diameter: determine the time of its oscillation; and the point within the sphere at which it must be suspended so as to oscillate in the same time.

94. Let a plane Isosceles triangle vibrate edgeways, suspended by its vertex. At what distance from its vertex must it strike an immoveable obstacle, so that its motion in the plane of vibration shall be destroyed.

95. Determine the centre of oscillation of an Isosceles triangle, vibrating about an axis which is perpendicular to its plane, and passing through

the vertex: and determine that, whose area being given, shall oscillate in the least time possible.

96. Three weights are placed at the angles of an equilateral triangle without weight, which is suspended by an axis perpendicular to its plane, bisecting one of the sides. Find the centre of oscillation.

97. Find the centres of oscillation of a square suspended by one angle, and oscillating *flat-ways* and *edge-ways*.

98. A gate of given weight and form is hung by hinges to a post inclined at a given angle to the vertical. When it swings freely, find the time of its small oscillations.

99. Determine the centre of oscillation of a cube vibrating about an axis coincident with one of its edges.

100. If AB be the diameter of a circle, whose centre is C, and CD be drawn perpendicular to the plane of the circle, which vibrates about an axis passing through D, parallel to AB. Determine the centre of oscillation.

101. If, in the case of the last problem, the circle move parallel to itself with its centre in the line CD, and thus generate a solid: determine the locus of A and B, so that every section of the solid perpendicular to CD may oscillate in the same time as a pendulum whose length is (l), the axis of suspension remaining the same.

102. If a plane circle oscillate on an axis which passes perpendicular to its plane, at the distance of half the radius from its centre; determine at

what other distance the axis may pass, so that the oscillation may be performed in the same time.

103. Determine the centre of oscillation of the circumference of a circle vibrating in a vertical plane; the axis of suspension being given.

104. Prove that a circular arc of given radius will oscillate through a given angle, in its own plane, about its middle point, in the same time, whatever be its length.

105. There is a point in the circumference of a circle, from which the circumference, which is supposed to be without weight, is suspended. Shew that if two equal weights be fixed at any points whatever in the circumference, equally distant from the point of suspension, and be made to vibrate in the plane of the circle, the time of oscillation will be equal to that of a pendulum, whose length is the diameter of the circle.

106. A semi-circular area is placed with its vertex upon a horizontal plane; determine the time of one of its small oscillations.

107. If the interior of two circles, which touch each other internally, be taken away, and the remainder vibrate edgeways round an axis passing through the point of contact; determine the time of an oscillation.

108. Determine the centres of oscillation of a common parabola vibrating *edge-ways* and *flat-ways*, when suspended by a given line from a given point above the vertex.

109. Determine the centre of oscillation of a

paraboloid, the axis of motion passing through the vertex: and also when its axis is parallel to the axis of motion.

110. Determine the centre of oscillation of a sphere, the axis of motion passing through a point in the surface.

111. Determine the centre of oscillation of a segment of a sphere; and also of the surface of the segment; the axis in each case being parallel to the axis of suspension.

112. Determine the centre of gyration of a circle revolving in its own plane about its centre: and also when it revolves about a diameter.

113. Determine the centre of gyration of the circumference of a circle; of the area of a circle; and of the surface of a sphere; the point of suspension in each case being in the circumference.

114. Determine the momentum of rotation of the plane of a circle revolving uniformly about one of its diameters; the density of the plane being as the $(n)^{\text{th}}$ power of the distance from the centre.

115. Having given the centre of gyration of a circle revolving about a diameter; determine the centre of gyration of an ellipse revolving about either axis.

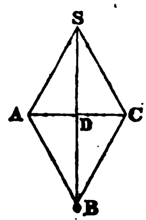
116. Determine the centre of gyration of a hyperboloid revolving about its axis.

117. Determine the centre of gyration of a sphere revolving about its diameter.

118. Determine the centres of gyration of a cone, and a paraboloid revolving about their axes.

119. What must be the solid of a revolution, so that when suspended by its vertex, the centre of oscillation may be in its base?

120. Let $SABC$ be a pendulum, in which SA, AB, BC, CS, are of the same kind of metal, and the cross bar AC of another kind, and such that the degrees of expansion, when of equal lengths, and exposed to equal additions of heat, may be very unequal, that of AC being the greater. Then if S be the point of suspension, and the several bars turn easily upon the points S, A, B, and C; determine the proportion of $SA : AC$, so that SB may remain constant.



121. A seconds' pendulum, of given length, in the form of a thin rectangular bar, suspended at the middle of its extremity by an axis perpendicular to its plane, being carried to the top of a mountain, the length of the bar was found to be diminished by a given quantity, in consequence of a change of temperature, the breadth remaining the same; and it lost (t'') in a day. Determine the mountain's height.

122. A straight bar, of given length, is made to oscillate in its own plane about an axis, situated in a line which bisects it at right angles. Determine the point of suspension, so that the time of an oscillation may be the least possible.

123. A heavy pendulum, in the form of a circular spindle, of given dimensions, is suspended from one extremity, and vibrates in an arc of

(a^0), at the rate of (n) times in a minute. Determine the horizontal force at its point of suspension when a maximum; and the position of the axis at that time.

124. A pendulum is composed of two thin wires, of equal length, at right angles to each other at the point of suspension, and vibrating in their own plane: determine the time of a small oscillation; and the angle at which they must be inclined to each other, so that the time of an oscillation may be doubled.

125. Into what curve must a rod of given length and uniform density be formed, so that performing its oscillations round an axis passing through its extremities, the time of oscillation may be a maximum?

126. When the force by which a watch balance is actuated, varies as the (n)th power of the distance from the point of the spiral spring's quiescence; determine the alteration in the daily rate, in consequence of a given change in the arc of vibration.

127. A lever, whose arms are inclined to each other at a given angle, and whose lengths and weights are respectively known, is made to vibrate flat-ways round an axis of suspension, which passes through the angular point of the lever. Determine the actual time of an oscillation.

128. Prove that the oscillations caused by the addition of a small weight to either scale of a balance are isochronous; and shew that the isochronous pendulum is equal to the diameter of the

circle which passes through the fulcrum and the points of suspension.

129. A heavy ring (W) hanging on a thread fastened at two points A and B, oscillates through a very small arc in the vertical plane of A and B. Determine the time of an oscillation, neglecting the magnitude of the ring and the inertia of the string.

130. A body (P) draws a lighter body (W) over a fixed pulley. A small oscillating motion is given to (W), at the commencement of (P)'s action. Determine the number of oscillations before (W) reaches the pulley:—shew that this number is independent of the string's length; and that however great (P) is, $\frac{1}{\sqrt{2}}$ oscillation at least will be performed in the time specified.

131. If two equal pendulums be put in motion at the same instant, one at the top, and the other at the bottom of a mountain (which is of the figure of a solid formed by the rotation of the catenary, and whose surface is equal to the square of the diameter of the base): and at the end of 24 hours, measured by the latter, a cannon ball be discharged, so as to describe the greatest range, and reaches the vertex at the same instant that the pendulum there has measured 24 hours: determine the height of the mountain, and the velocity with which the ball is discharged.

132. A string bearing a weight (P) at its extremity, is just strong enough to support it after oscillating through 60°. Shew that the angle (θ),

through which it may oscillate, so as just to support a weight (W), may be determined from the equation:

$$\theta = \text{versin.}^{-1}\left(\frac{P}{W} - \frac{1}{2}\right).$$

SECTION XI.

1. LET two weights (P) and (W) be hung at the ends of a straight lever whose arms are (a) and (b). Determine the pressure upon the fulcrum while the lever is descending from a horizontal to a vertical position, and also the velocity of the motion; the inertia of the lever being neglected.

2. Let the axis of motion pass through a thin straight rod of uniform thickness and density, dividing the rod into two arms that have the ratio of 2 : 1. Determine the velocity acquired by the extremity of the longer arm, when the rod shall have moved from a horizontal to a vertical position.

3. Let a cylinder begin to move from a horizontal position round one of its ends, which remains fixed upon a fulcrum. Compare the pressure on that end at the beginning of the motion with the whole weight of the cylinder.

4. When (W) is raised by (P) in the wheel and axle; determine the pressure on the axis, neglecting the inertia of the machine.

5. If a weight (P) raise another (W) by means of the wheel and axle, whose radii are (r) and (r'); find the space descended in (n''); neglecting the inertia of the machine.

6. Let a weight P fastened to a string going over a wheel, by its descent cause two weights Q, Q', to be wound up on two axles. Determine the velocity of P, after that it has descended through a space (s), the radii of the wheel and of the two axles being r, r', r''.

7. If a weight (P) raise another (W) by means of the wheel and axle; having given (P) and (W) and the radius of the wheel, determine that of the axle, so that the axis may sustain the least possible pressure.

8. Materials are to be raised through a given altitude by a wheel and axle whose radii are known; the power, which is given, being applied to the circumference of the wheel. Determine the quantity raised at each ascent, when the greatest quantity in the whole is raised in a given time; the inertia of the machine being neglected.

9. In a combination of two wheels and axles, the circumference of each wheel is (n) inches, of each axle (1); and a weight P is applied to the circumference of one of the wheels to raise matter to a certain height. How much must be raised each time, that the whole quantity may be raised in the least time possible?

10. The axis of a wheel and axle is placed in a horizontal position, and a weight (w) which is applied at the circumference of the axle is raised by the application of a given moving force (p) applied to the circumference of the wheel. Given the radii of the wheel and axle; to determine (w) when the moment generated in it in a given time

is a maximum, the inertia of the wheel and axle not being considered.

11. Let a weight appended at the circumference of a wheel elevate by its gravity a weight appended at the axle. Given the two weights, the weight of the wheel and the radius of the axle; to find at what distance one of the weights must be applied, so as to raise the other at the axle through a given space in the least time.

12. If the weight of the wheel and axle be (w); and the axis be horizontal; having given a weight (w') applied at the circumference of the wheel, and the weight (w'') applied at the circumference of the axle: determine the velocity of the descending weight (w') at the end of (n) seconds.

13. If (w) and (w') be two weights applied at the circumferences of a wheel and axle; determine the proportion of the radii, so that the time of (w') ascending through a given space may be a minimum, the inertia of the wheel and axle being considered.

14. A power P draws up a weight W by a wheel whose breadth is just sufficient to admit one coil of a rope, the thickness of which is $2r$, so that the rope perpetually coils on itself. Neglecting the excess of weight of the rope on one side of the wheel over that on the other, determine where W has acquired its greatest velocity.

15. To two wheels of unequal radii, whose weights are known and supposed to be condensed into their circumferences, two equal axles are attached, from which are suspended two equal

weights. Determine how long these wheels, which are supposed to move freely on their axles, have been in motion when the weight attached to one axle has descended (n) feet farther in the same time than the other.

16. In a system of (n) equal wheels and axles, where (W) is raised by a power applied at the circumference of the first wheel; determine the force accelerating (W), and the tension of the string to which (W) is attached; (w) being the weight of each wheel with its axis, (d) the distance of the centre of gyration from the axis, and $r : 1$ the ratio of the radii of each wheel and axle.

17. If (P) and (W) be two bodies hanging over a fixed pulley, and (P) descend; determine the space descended in (n''); and the velocity acquired.

18. Two bodies are connected by a cord passing over a fixed pulley; determine their ratio, so that one of them shall generate the greatest quantity of motion possible in the other in a given time.

19. Two bodies (P) and (W), connected by a string passing over a fixed pulley, are in the ratio of $n : 1$. Determine the space through which a heavy body will descend by the action of gravity in the time that (P) descends (a) feet.

20. Two bodies (P) and (W) support each other on a single pulley: if a weight (w) be added to (P), determine how long it is in descending through (n) feet; and what velocity it has acquired.

21. Two equal weights (P) and (W) are connected by a string passing over a fixed pulley; what weight added to (P) will cause it to acquire in (n'') a velocity (v). And through what space will it have descended in that time?

22. If a power (P) descending vertically by means of a string passing over a fixed pulley, elevates a weight (W); determine what will be the pressure sustained by the axis of the pulley.

23. If in a system of moveable pullies, each hanging by separate strings, the weight be double the power; determine the number of moveable pullies, so that the power may descend through four feet in the first second.

24. With what weight must a given weight (P) be connected by a string passing over a fixed pulley, so as to describe the same space in a given time as when it descends freely down a given inclined plane?

25. A given quantity of materials is to be raised by a given weight (P) passing over a fixed pulley. Determine the quantity to be raised each time, so that the time of raising the whole may be a minimum. Determine also the quantity to be raised each time, so that the greatest quantity may be raised in a given time.

26. A weight (P) raises up another weight (W) by means of a string passing over a fixed cylindrical pulley; compare the tensions of the two parts of the string; the weight of the string and the friction at the axis of the pulley being neglected.

27. A weight is fixed to the lowest point in a circle moveable in a vertical plane about its centre; another equal weight is attached to a string wrapped round the circumference. Determine the velocity acquired by the descending weight in any space.

28. Two weights (P) and (W) are connected by a string passing over a fixed pulley. Determine the velocity of P at any point of its descent from rest, and also the time of descent; the weight of the string and pulley being considered.

29. Two equal weights are connected by a string passing over a fixed pulley. Supposing a weight to be added on one side, and the length and weight of the string and the difference of the altitudes of the weights at the commencement of the motion to be given; determine in what part of the descent the velocity will be neither increased nor diminished by the weight of the string.

30. In a system of two pullies where each string is attached to the weight, (P) draws up (W). Determine the accelerating force on (P), the tensions of the strings, and the pressures on the centres of the pullies; the weight and inertia of the pullies being considered.

31. Two equal weights are connected by a string of uniform density and thickness which passes over a fixed pulley, and the string when suspended freely will just support a weight (W). The whole being put in motion, find the time elapsed before the string breaks.

32. A quadrantal arc is placed in a vertical plane with one of its bounding radii horizontal; and at the highest point of the arc is placed a pulley, over which passes a string supporting two weights (P) and (W); of which (P) is double of (W) and hangs freely, (W) being supported on the arc. Determine the time in which (W) will be drawn up from the bottom to the top of the arc by the descent of (P); and what will be the last velocity.

33. If two given weights be fastened to the ends of a string which passes over a fixed pulley, and the greater weight descend by the force of gravity and draw up the other along a curve which is in the same vertical plane as the pulley. Determine the nature of the curve when the time of ascent from one given point to another is the least possible.

34. If a body be drawn up a cycloidal canal placed with its axis vertical, by means of an equal weight passing over a pulley at the highest point of the arc; shew that the time of ascending to the highest point of the arc is to the time of half an oscillation as $1 : \sqrt{2}$.

35. A weight (W) is raised upon a moveable pulley. The two extremities of the cord are wound in different directions about two cylinders which have a common axis but different radii; and the power (P) descends unwinding a string from a wheel of given radius upon the same axis. What is the force which accelerates (P)'s descent, when the strings are parallel to each other?

36. Let (P) and (W) be two bodies connected by a string passing over a fixed pulley; of which, (P) hanging freely descends vertically and draws (W) along a horizontal plane. Determine the velocity acquired by (W) whilst it is drawn by (P) through a given space: find also the velocity acquired by (P) and the space described in (t'').

37. AB, BC, are two given inclined planes, having the obtuse angle ABC contained between them equal to 120°, and the point C resting on a horizontal plane. What weight P acting over a fixed pulley at A, will acquire, in drawing a given weight W from rest up BA, the same velocity that would be acquired by a heavy body falling from rest through both the planes.

38. The weight (P) after descending freely through (a) feet begins to draw up another greater weight (W) by a string passing over a pulley. Determine the greatest height to which (W) could rise, and the time of rising.

39. A given weight (P), by means of an extremely thin string passing over a fixed pulley, draws up a chain of an indefinite length loosely coiled up on a horizontal plane directly below the pulley. Determine the velocity acquired by (P) at any point of its descent; and also the point where it will begin to reascend.

40. If two weights be suspended at the ends of two strings passing over two fixed pullies, and having their other ends attached to the extremities of a given rod which is moveable about a fixed point; determine the velocities of the

weights when they move in a vertical direction; the weight of the rod being considered.

---

41. Let a weight P descending vertically draw W up an inclined plane whose angle of elevation is 30°; determine the velocity of P, after (n) seconds have elapsed.

42. If a weight (P) descending vertically draw (W) up an inclined plane by means of a string passing over a pulley fixed at a given height above it; determine the velocity acquired by (P) in describing a given space.

43. Determine the length of an inclined plane, when a weight (P) acting parallel to the length elevates a weight (W) through the length in (n) seconds, the angle of inclination being 30°.

44. Two bodies are connected by a string and sustained upon two inclined planes of equal altitudes, by a string passing over a fixed pulley, and acting in a direction parallel to the planes. Determine how far one will descend in (t''), and the velocity acquired.

45. Two bodies (P) and (W) are connected by a string passing over a pulley at A, the vertex of two inclined planes of the same altitude: given the plane AC, to determine the elevation of AB, such that (W) shall be drawn by (P) from the horizontal line BC to the point A in the least time possible.

46. Two weights, which are to each other as

the length and height of an inclined plane, hang over a fixed pulley; and it is observed that the heavier weight descends down (a) feet to acquire the same velocity which the body on the plane acquires in descending through (b) feet; and the base of the plane is $(\frac{1}{n})^{\text{th}}$ of the length and height. Determine the weights.

47. A given weight (P) draws another given weight (W) up an inclined plane of given height and length: determine when and where (P) must cease to act, that (W) may just reach the top.

48. Given the height of an inclined plane, to determine its length, so that a given power shall raise a given weight up it in the least time possible.

49. Let (L) be the length of an inclined plane, inclined to the horizon at an angle of 45°: compare the time down it with the time of an oscillation of a pendulum whose length is (L).

50. A weight (P) descending vertically draws another weight (W), which is equal to $(\frac{2}{3}P)$, up an inclined plane, whose elevation is 30°. How often will a pendulum, whose length is (l) inches, vibrate in the time that (P) descends (a) feet, the length of the common pendulum being known?

51. If a given weight (W) descend from rest along a given inclined plane, and by means of a thread passing over a pulley, draw another weight (W') through a given space up another given inclined plane: determine the weight (W') when its momentum thus acquired is a maximum, the thread acting parallel to the plane.

52. If a weight (P) draws another (W) up a groove cut out in an inclined plane: determine the velocity of (W) at any point, the angle which the string makes with the plane varying at every point.

53. A given weight (P) is attached to a given cylinder by means of a string wrapped round its circumference, and passing over the common vertex of two inclined planes; (P) is drawn up one while the cylinder descends down the other. Compare the lengths of the planes.

54. A body whose weight is (W), falls down the length of an inclined plane, which has the power of moving freely along a horizontal plane on which it stands. Having given the weight of the prismatic figure composing the plane; determine the path of the body (W), the time of describing it, and the last acquired velocity of the moveable plane along the horizontal plane.

55. A known weight (W), at the extremity of a rod which passes through two small rings fixed in the same vertical line, by its pressure puts a solid inclined plane in motion along a horizontal plane. Having given the weight of the plane; determine its elevation, so that the velocity communicated to it in a given time may be a maximum.

56. If one end of a block AB rest upon a horizontal plane ACD, and the other be supported by a wedge CDE, in the form of a right-angled triangle; the length and weight of the block, as also the base CD, and perpendicular DE, of the

wedge being given: determine what force must be applied to the back of the wedge, in a direction parallel to the horizon, to keep the block from sliding; a given part CF of the base of the wedge being introduced beneath it, and the extremity A being fixed so as to be prevented from sliding.

57. If two given bodies, (A) and (B), connected by a string, be supported on an inclined plane by means of a string passing through a ring at A, and any given force be impressed in a horizontal direction on (A); determine the nature of the curve described by the body.

58. A given weight (P) is connected with a cylinder, by means of a string wound round it, and descends through (a) feet in (n) seconds, causing the cylinder to revolve round its axis. Determine the weight of the cylinder.

59. A bucket descends into a well, unwinding a string from a cylinder of given weight and radius. What is the velocity acquired in falling through a given space, and the time of descent, the weight of the string being neglected?

60. A cylinder of given weight and dimensions is put in motion round an axis parallel to the horizon, by a given weight (P) suspended by a small string wound round the surface of the cylinder: determine the actual time in which P would descend from the surface of the earth to the centre.

61. A cylinder of given weight and radius revolves about its horizontal axis. Determine the time in which a weight (W) acting by means of a string at the circumference of the cylinder, will

generate a velocity of one foot *per* second at a distance (1) from the axis.

62. If a cylinder revolve round a vertical axis; and about it there be wound a string which passes over a fixed pulley, and has a given weight attached to it: supposing the centre of gravity of the system to be in the axis; determine the radius of the cylinder, so that the angular velocity generated in a given time may be a maximum.

63. Determine the space described in a given time by a given heavy sphere rolling down a given inclined plane.

64. A cylinder revolves down an inclined plane, having its axis always horizontal: compare the force which accelerates the axis with gravity.

65. Determine what must be the inclination of a plane of given length, so that a heavy hollow cylinder of known dimensions and uniform thickness may roll freely down it in (n) seconds.

66. If a heavy hollow cylinder of uniform thickness roll freely along an inclined plane of given length and elevation, in a given time; knowing the internal diameter, determine the thickness.

67. If a solid cylinder, and a thin hollow cylinder of the same *weight* and radius roll together from rest down a given inclined plane; determine how far they will be separated after a given time.

68. Two equal solid cylinders are connected by a string without weight, wrapped round each and passing over a fixed pulley. Find the pressure on the centre of the pulley whilst they both descend.

69. A string wrapped round a *hollow* cylinder

of uniform density, whose radii are R and r, passes over a fixed pulley, and has a weight attached to it. Determine the space descended by the cylinder in a given time.

70. A *hollow* cylinder of known weight is supported with its axis horizontal; and a given weight (P) connected with the cylinder by means of a thin string wound round it, descends through a given space in (n) seconds: having given the external determine the internal diameter of the cylinder.

71. A *hollow* cylinder of known weight is placed with its axis horizontal; and a string of given length is fixed to and wound round it; to the extremity of which a weight (P) being attached, is suffered to descend. Determine the height to which (P) rises on its *ascent*.

72. A flexible string of given length is wrapped round a cylinder, whose weight and dimensions are given; one end of it being fixed to the surface of the cylinder, and the other to a tack; if the cylinder be suffered to descend by its gravity, determine the time in which the whole string is unwound, and the velocity acquired.

73. A perfectly flexible chain is wound round a cylinder, supported with its axis parallel to the horizon. Having given the weight and dimensions of the cylinder, and also the length and weight of the chain; determine the time in which the chain, impelled by the force of gravity, will unwind itself; a given length being unwound at the commencement of the motion.

74. The weight (P) is attached to a string,

which passes over a fixed pulley, and is wound round a thin cylinder, whose weight is (W). Determine (P) such that it shall neither ascend nor descend, whilst the cylinder descends by the unwinding of the string.

75. Given two weights, (W) and (W') connected by a string passing over a pulley at A, and an inclined plane AC moveable about an axis passing through A. The lesser weight (W') being placed upon the plane at a given distance from A, whilst (W) hangs freely; determine the velocity of the plane, so that (W) may neither ascend nor descend.

76. A string passing over a fixed pulley is coiled, on each side of it, round two cylinders of equal weight (W), the one being of uniform density, the other collected in the circumference: determine the tension of the string when they are at liberty to move; the inertia of the string and pulley not being taken into account.

77. A sphere whose weight is (W) is made to roll down an inclined plane, by means of a string wrapped round it, which passes over a pulley so as to be kept parallel to the plane, and has a weight (P) which is equal to $\left(\frac{W}{n}\right)$ attached to it. Determine the proportion of the height of the plane to its length, that (P) may neither ascend nor descend.

78. A sphere and a hollow cylinder of equal weight are suspended by a string passing over a solid cylindrical pulley equal in weight to the

former. Determine the circumstances of the motion, and the tension of the string.

79. A cylinder of given dimensions rolls down the convex surface of a given hemisphere. Determine the point where it will leave the hemisphere, and the time of its descent from the top by the force of gravity.

80. A body having fallen from a given altitude, begins to descend, with the velocity acquired, along the arc of a given cycloid, whose base coincides with a horizontal plane: determine where the body will leave the cycloid, and where it will meet the horizontal plane.

81. A body descends down the convex side of a logarithmic curve, placed with its asymptote parallel to the horizon. Determine where it leaves the curve.

82. A hollow sphere, whose external and internal radii are known, *rolls* down a given inclined plane. Determine the inclination of another plane of the same length, so that it may *slide* down it in the same time.

83. A weight (*P*) is *supported* by a string, which passes over a fixed pulley, is wound several times round a cylinder (*Q*), and attached to a pin on the other side, the strings being all parallel. If the string be now cut between the cylinder and this point of support, determine the motions of (*P*) and (*Q*.)

84. If a given segment of a sphere, greater than a hemisphere, touching a horizontal perfectly polished plane, with its surface at the edge of the base, be thus put in motion by the force of uni-

form gravity; how far will it move along the plane, and what will be its velocity?

85. Determine the velocity of *any* segment of a sphere rolling along a horizontal plane, such as may keep it (if possible) on a given circle parallel to the base.

86. If the vertex of a given cone rest on an inclined plane, whilst the axis is in a vertical direction; determine the velocity of the centre of gravity of the cone when it is suffered to descend by the force of gravity, the vertex being at liberty to slide freely along the inclined plane.

87. Determine how long a heavy cylinder, of given radius, will be in motion, and how many revolutions it will make about its axis in descending by its gravity down a plane of given length and inclination; supposing it to be found by experiment, that if the plane's inclination were n^0, the friction would be just sufficient to cause the cylinder to roll down without sliding, and that when a body slides along a plane, the friction is as the pressure against the plane.

88. If a chain of given weight, reaching to the centre of the earth, be suspended from a cylinder at the surface, round which it is made to wind itself by the descent of a weight (W) equal to the weight of the chain, unwinding a string supposed to be without weight. Determine the velocity of W at any point; and also when it is the greatest, and when it is nothing.

89. If a given pendulum be suspended on a pin fixed in the centre of gravity of a given vessel resting on a horizontal and perfectly smooth

plane; determine how far the pendulum descending from a horizontal position will move the vessel during one whole vibration; and the locus of the weight of the pendulum; the weights of the vessel and pendulum being given, and the distance of their centres of gravity.

90. If the pendulum and vessel be put in motion by the force of uniform gravity; determine their velocities and the time of vibration.

91. Determine the quantity and direction of the force exerted by a door of given weight and dimensions on each of its hinges.

92. If a door of given width be set open in a situation perpendicular to its position when shut, in what time will the door be quite shut by a weight (w) drawing it by its extreme side, by a cord passing over a pulley fixed in the door frame: and what will be the last velocity of the weight and door, supposing its weight and friction on its hinges to be such that a force of (n) pounds weight is just in equilibrio with it in its first position?

---

93. If a ring begin to descend freely from the extremity of the horizontal radius of a given quadrant down the arc whilst another begins to descend freely down its chord; determine the places of the rings, and the times when they are in the same vertical line.

94. Determine the velocity acquired by the middle point of a rod in falling from a vertical to a horizontal position, the bottom being prevented from sliding.

95. A given cylindrical rod falls by gravity to-

wards a horizontal plane, whilst at the same time its extremity moves freely along the plane. Determine the pressure upon the plane in any position, and the velocity of the moving point.

96. A rod is placed in an inclined position with one end upon a perfectly smooth horizontal plane. Determine the equation of the curve described by the other extremity whilst it falls.

97. Two bodies (P) and (W) connected by an inflexible rod, and acted on by gravity, move in the circumference of a vertical circle. Determine the tension of the rod in any position.

98. A given uniform rod moves in the same plane in a hemisphere. Determine its motion.

99. A rod of uniform thickness and density is inclined against a wall upon a perfectly smooth horizontal plane. Determine the velocity acquired by the middle point of the rod in the descent; and shew that this velocity is to the velocity of a body falling freely down the same perpendicular height :: $\sqrt{3} : 2$.

100. Two bodies (A) and (B) are placed upon a horizontal plane and connected by a rigid rod without weight. If a body (C) impinges upon a given point of the rod, in a given direction, and with a given velocity; determine the motions of (A) and (B), the body (C) not being connected with the system after the impact.

101. If an uniform slender rod AB parallel to the horizon, and supposed without weight, have two equal bodies fixed to it, one at each end, and revolve round a point C in that rod as a centre in any given time; having given the length of the

rod, and the distance of the middle of the rod from the centre of motion C; determine the velocity with which the rod passes through the point C; as also the time.

102. Two bodies connected by an inflexible rod without weight, and to one of which a certain velocity is communicated, are constrained to move along two grooves at right angles to each other. Determine the circumstances of motion; and shew that when the bodies are equal, the line which joins them revolves uniformly.

103. (A) and (B) are two material points connected by an inflexible rod without weight; B moves on the horizontal plane CB; (A) descends along the inclined plane AC, the motion of the rod being in a vertical plane. Compare the velocity which (A) has at the point C with the velocity which it would have if it were to descend freely down AC.

104. (A) and (B) are two material points connected by an inflexible line AB; (A) moves along a groove, and AB on a smooth horizontal plane. Having given the initial position of the rod, and the quantity and direction of the motion communicated to (A); determine the angular velocity of the rod when it coincides with the groove.

105. Two equal bodies (A) and (B) are connected by a string of given length: (A) is placed in a horizontal groove, and (B) hangs freely down, the string passing through an aperture which is continued along the bottom of the groove; a given velocity being communicated to (A); find the position of (B) at the end of (t).

106. If one end of a string be fixed to the extremity of a body placed on a perfectly polished plane, and any force be impressed upon the body; it is required to determine its motion.

107. A chain of given length, consisting of indefinitely small equal links, being laid straight on an horizontal perfectly polished plane, except one part of given length which hangs down vertically from the plane: determine in what time the chain will entirely quit the plane.

108. A chain, whose length is l, is placed along an inclined plane, whose height is (n) and length (m), so that one end may coincide with the lowest point of the plane. Shew that the whole time of the chain's sliding off the plane $=$

$$\sqrt{\frac{m\,.\,l}{m-n}} \times \text{h}\,.\,\text{l}\left(\frac{m+\sqrt{m^2-n^2}}{n}\right).\,g;\ g \text{ being} = 32\tfrac{1}{6}.$$

109. A chain of given length consisting of indefinitely small links, being put over a pulley void of friction; and the two parts depending from the pulley being unequal: determine in what time the chain will fall from the pulley.

110. (a) and (b) are two fixed pullies in the same horizontal line, over which a string passes, to which (m), (m'), (m), three equal weights are suspended. Suppose (m') to be let fall from c, the middle point of ab, determine the distance from c, at which its velocity is the greatest.

111. If two given equal semicircles in the same vertical plane be placed with their diameters in the same horizontal line, whilst two equal weights connected by a string are put in motion up the semicircles, beginning at the lowest points, by

the gravity of a third weight hung in the middle of the string, which at first is horizontal, and slides freely over two pullies placed in the nearest angles of the semicircles. Determine the ratio of the third weight to each of the equal ones, so as to be just able to raise them to the pullies; and also the tension of the string when in motion.

112. If a string of given length be fastened at one end to a given point on a smooth horizontal plane having a straight edge, and a weight be attached to the other end of the thread and brought to the edge of the plane with the string stretched; then if the weight be suffered to descend by the force of gravity from this position; determine the velocity of the descending weight at any time, and the curve which it describes.

113. If a flexible string, of given length and without weight, be fastened at one end to a point given in position, and a small heavy body slide freely thereon; determine the locus of the body, whilst the other end of the string is carried slowly along a right line given in position in the same vertical plane with the given point.

114. Two equal bodies are placed on a smooth horizontal plane, and connected by a flexible string which is stretched out in a direction parallel to the edge of the plane, and having to its middle point another string fastened, which bears at its extremity a given weight hanging over the edge of the plane, and by its descent communicating motion to the other bodies. Having given the weights of the bodies and the length of the

connecting string, determine the motion of the descending weight, and its place at the end of any given time.

115. Two bodies of known weight, lying on a smooth horizontal plane, are connected by a flexible string passing through a small ring fixed at a given point between them. In this position a given velocity is communicated to one of them in a direction perpendicular to the line that joins their centres, and the other is made to move directly towards the ring. Determine the motion of the projected body; and find the angle described when the other body arrives at the ring.

116. Two bodies, of known weight, are connected by a flexible string passing through a small hole in a perfectly smooth horizontal plane, and the body which is above the plane is projected upon it with a given velocity at a given distance from the hole, in a direction perpendicular to the string, which is kept tight by the other body hanging freely. Determine the velocity, time, and angle, formed by the string with the first position when the projected body has arrived at any other distance from the hole through which the string passes.

117. An uniform chain is coiled on a smooth horizontal plane, and a given length being drawn out is projected along the plane with a given velocity. Determine the velocity after the description of any given space.

118. *ACBDG* a perfectly flexible chain of given length being fastened at *A*, passes over the pulley *B*, which is placed close to *A*. The ex-

cess (DG) of BG above BC at the commencement of the motion being given, determine the velocity of the chain after a given portion of it has been drawn over the pulley.

119. Determine the position in which a given flexible line must be held, one end of which is fixed to an immoveable tack; so that a heavy body fastened to the other end, after descending freely from rest till the line becomes stretched, may ascend to the greatest height possible on the other side: determine also that height, together with the whole arc described, and the point where the body quits the circumference of the circle.

120. In what direction must a ball be projected along a hollow spherical surface, so that it may pass through a given point; the ball being supposed to be without weight.

121. A quadrant being placed vertically, a chain of given weight is laid on the arc, and coincides with it. Determine the force necessary to keep it from sliding down.

122. If a perfectly flexible and uniform chain, of a given weight, coincide with the convex surface of a vertical quadrant having one radius horizontal; determine the velocity acquired in its descent, and the tension at a given point in any given position of the chain.

123. On a smooth cycloidal lamina, whose vertex is upwards, is laid a chain having its upper extremity on the vertex. Determine the time of its running down the whole curve, and the velocity acquired.

124. A perfectly flexible chain of uniform thickness, is suspended from two given points: determine the law of its density and weight, so that it may form itself into a parabola.

125. A chain of uniform thickness hanging freely, forms itself into a cubical parabola. Determine the law of its density and weight.

126. Determine the law of weights, which press each particle of a perfectly flexible line in such a manner as that it shall form a curve whose equation is $a^3x = y^4$.

127. Given the length of a chain, and the law of its density such that if its ends are fixed to two points in the same horizontal line, at a given distance from each other, it will dispose itself into a circular form; determine the law of the density, and the weight of the chain, supposing one foot of either end to weigh one pound.

128. An uniform elastic string being of such a length, that when it hangs vertically if an equal quantity were appended to the lowest point it would stretch it to twice that length; what weight must be appended at the middle point that the increase of length may be three-fourths of the original.

129. If an elastic cord of uniform density, whose length is (L), and weight (W), be stretched in a horizontal position by a given weight (w), and the increment of length be (l): determine the length of the cord when suspended by one of its extremities; the increment of its length being always as the weight which stretches it.

SECTION XII.

1. Let a system of bodies be moveable round a vertical axis. At what distance from this axis must a given force act, so that the angular velocity communicated to the system in a given time may be the greatest possible.

2. If a leaden ball of given diameter strike perpendicularly a pendulous cylinder of oak, and impel it through an angle of 60°, the radius of the arc being (r). Determine the velocity of the ball, and the cylinder's greatest perpendicular force on the centre of suspension; the whole solidity being given, the axis of motion being in the vertex, and the ball supposed to strike the cylinder (a) inches from the lower end.

3. An isosceles right-angled triangle ABC is suspended at the right angle A, and its side AB, which is equal to $4a$, is kept in a vertical position by a ring at B. An angular velocity (v) being communicated to the triangle round AB, shew that there will be no pressure at B, if $v^2 = \frac{g}{a}$.

4. Determine the greatest angular velocity that can be acquired by the lever ACB, whose arms CA, CB, are equal, and at right angles to each other, CA being at first horizontal, and the density of the arms varying as the distance from C.

5. Let ABC be a thin triangular plate of heavy

metal; and from B let BD be drawn to meet AC in D, so that BC being greater than AB, AD may be greater than CD. Determine the velocity with which the triangle must revolve about BD as an axis, so that BD may remain vertical, the angular point B resting on a horizontal plane.

6. Determine in what manner a ring will move upon a straight rod revolving about one of its extremities on a horizontal plane, when after the first percussive impulse, they are disturbed by no force but their mutual pressure against each other.

7. If a given right-angled plane triangle, whose hypothenuse is an uniform slender rod, revolve about its perpendicular as an axis, whilst a ring slides freely along the hypothenuse; determine the time of the ring's descent down the hypothenuse, its length and the perpendicular being given, and the time of one revolution round the axis being (t'').

8. If the arc of a quadrant revolve about its vertical radius with an uniform velocity, whilst a ring at the upper end is left to descend by its own weight: having given the radius, and the time of a revolution, determine in what time the ring will arrive at the lowest point.

9. A thin rod, in the form of a quadrantal arc, revolves round an axis perpendicular to the horizon; and a ring moves freely on the rod. Determine with what velocity the highest point of the arc must revolve, that the ring may always remain in the middle of the arc.

10. A body urged by gravity descends in the quadrant of a circle, and is at the same time acted upon by a repulsive force placed in the lowest point, and varying inversely as the square of the distance. Determine the velocity of the body at any point of its descent; and its positions when at rest; and also when its velocity is the greatest.

11. A parabola revolves round its axis which is vertical, in a given time, and the angular motion will just prevent a body at any point of the curve from descending. Determine the parameter of the parabola.

12. If a parabolic rod revolve round an axis perpendicular to the horizon, and a ring be placed upon it; then if the parabola revolve about its axis with such a velocity that the ring may remain at rest, it will remain at rest in every other situation.

13. If a parabolic rod AVB, whose vertex is V, revolve about a double ordinate AB, with a given angular velocity, and a ring be at liberty to slide down from A; determine the lowest point to which it can descend, and the time of descending.

14. A ring of given weight descends by its gravity down the arc of any algebraic curve, and the curve revolves uniformly about its axis, which is vertical in (t''). Determine the velocity of the ring at any point of its descent.

15. If a slender rod of given length be fixed perpendicularly to the earth's surface, in a given latitude; from the top of which a ring of heavy

metal descends by its own gravity: determine how long it will be before it reaches the surface of the earth; supposing the force of gravity uniform, and the earth to revolve round its axis in 24 hours.

16. Two chains of the same uniform thickness and density, are suspended from two given points, and attracted towards a centre of force, the law of the force being any power or root of the distance. Shew that the pressures on the points of suspension are proportional to the squares of the velocities which would be acquired by bodies falling towards the centre from the points of suspension down spaces which are equal to the lengths of the chains.

17. Determine the nature of the curve into which a flexible wire of given length must be bent, so that a ring of heavy metal being put on it, and the wire revolving about a vertical axis with a given velocity, the ring may rest in equilibrio, the abscissa being to the ordinate as $m : n$.

18. A small ring slides freely upon a string of given length, whose ends are fixed to the extremities of a given rod: determine the equation of the curve described by the ring whilst the rod revolves gently round one end in a vertical plane.

19. If the ends of a perfectly flexible string of given length be fixed to two given points, and a small heavy body descend freely along the string by the force of gravity: determine the time of descending through a given portion of the string.

20. If a weight having a rod of (n) feet long fixed to its bottom, descend from a height less than the length of the rod, the lower end of which slides freely along a horizontal plane; and a ring be placed on the rod touching the weight when it first begins to descend. Determine from what height the weight should fall, so that when it reaches the horizontal plane, the ring shall have run over $\frac{1}{m}^{\text{th}}$ of the rod; there being no resistance to the body's fall, either by the rod, which is considered without weight, or the ring, which is considered very small.

21. Two equal heavy balls are suspended by wires, of the same given length, from the vertical axis of a machine, and are just in contact. How far will they separate from one another when a given angular velocity is communicated to the system?

22. If two given weights, (W) and (W'), begin at the same instant to slide freely from A down the arms of a bent lever BAC; determine the paths they will describe, and the times of their quitting the lever; the length of the arms being given, and the lever, supposed without weight, to move freely about the centre A.

SECTION XIII.

1. The initial force accelerating a body down a circular arc, is to the force accelerating it down the chord :: 2 : 1, ultimately.

2. Determine the nature of the curve, such that a body beginning its motion from a given point *A* with the velocity acquired down a given line *BA*, may afterwards recede uniformly from *AM*, which is parallel to the horizon.

3. Determine the nature of the curve, along which a heavy body descending by the force of gravity will press upon the curve at any point with a force proportional to the ordinate at that point.

4. Determine the nature of the curve, along which a heavy body descending by the force of gravity with a given initial velocity, shall press upon the curve at any point with a force reciprocally proportional to the radius of curvature at that point.

5. Determine the curve, in which a body must move, so as to continue always at the same invariable distance from another body moving uniformly in a right line; the velocity of the former being also uniform, and exceeding that of the latter in any given ratio.

6. The velocities of two bodies (*A*) and (*B*)

are in a given ratio, and they begin to move at the same time from A and B the extremities of a given line AB: (A) moving uniformly in a direction inclined at a given angle to AB, and (B) uniformly in the direction BA. Determine the nature of the curve, to which the line joining the bodies is always a tangent.

7. A body (P) moves uniformly along a straight line, and a body (Q) in pursuit of it moves always directly towards it with a velocity which is to that of (P) as $1 : n$. Determine the nature of the curve described by (Q).

8. A body describes the quadrant of a circle touching a vertical line at its highest point, being acted upon by a force perpendicular to the horizon. Determine the law of the force which will make it recede uniformly from the horizontal radius, and the time elapsed, and the velocity acquired at any point of the descent.

9. A body acted on by gravity is made to ascend along the concave part of a vertical semicircle from the extremity of the horizontal radius; determine its initial velocity so that after quitting the circumference it may pass through the centre.

10. Determine on a vertical plane the curve AP such that the whole perpendicular pressure on any point P, caused by a body falling freely from A down the curve, may vary directly as the $(n)^{th}$ power of the vertical height through which it has descended.

11. A body descends down the arc of a parabola, whose plane is vertical and axis horizontal;

and is acted upon by gravity and a repulsive force tending from the focus and varying inversely as the $(n)^{th}$ power of the distance. Determine the velocity of the body at any point of its descent.

12. If S and S' be two centres of force, which varies inversely as the squares of the distances; and their intensities be equal. Determine the path described by a body, and its velocity at any point in that path: supposing it to begin to descend from a point which is at an equal distance from the two centres of force.

13. If the distance of the point from which the body begins to descend, as in the last problem, had been at an inconsiderable distance from the straight line joining S and S'. Determine the time of the body's oscillation.

14. Two material points (S) and (P), the mass of the first being double that of the second, attract each other with a force which varies inversely as the square of the distance. When they have approached each other by half their original distance, (P) receives a new perpendicular impulse, which communicates to it a velocity equal to that which (S) has acquired. What curve will now be described by each about the other?

15. If in a given latitude a stone be let fall from a man's hand into a hole in the earth, so as to fall freely without touching the sides: determine the nature of the curve in which the perforation must be made; the greatest velocity of the descending stone; the time of descent, and the nearest approach to the centre, allowing for

the earth's diurnal motion round its axis, supposing it to be a sphere, and the perforation a perfect vacuum.

---

16. Of all semiparabolas having the same area, required that along which a heavy body will descend by the force of gravity in the least time possible.

17. Determine the nature of the curve in which a body descends from one given point to another in the least time possible; the velocity at each point being supposed to vary as the corresponding ordinate of the curve.

18. How must the force of gravity vary, when a circular arc is the line of swiftest descent from one given point to another?

19. A point and a straight line being given in position in the same vertical plane; and a heavy body acted upon by gravity descends from the given point in a circular arc to the given line in the shortest time possible; determine the length of the arc.

20. Prove that an arc of a circle which does not exceed 60° is a curve of quicker descent than any other curve which can be drawn within the same arc; and the arc of 90° is a curve of quicker descent than any other curve which can be drawn without the same arc.

21. In what curve must a body be constrained to move upon the surface of an upright cylinder, so that when acted upon by gravity it may de-

scend from one given point to another in the shortest time possible?

22. If A and P be two points in the same vertical plane, and AP be joined, meeting in Q an inverted cycloid AQB, whose base AB is part of the horizontal line AX; and QB be joined, parallel to which PC is drawn meeting AX in C; then will AC be the base of an inverted cycloid, through which a body will pass from A to C in the shortest time possible.

THE END.

LONDON:
PRINTED BY R. GILBERT, ST. JOHN'S SQUARE, CLERKENWELL.